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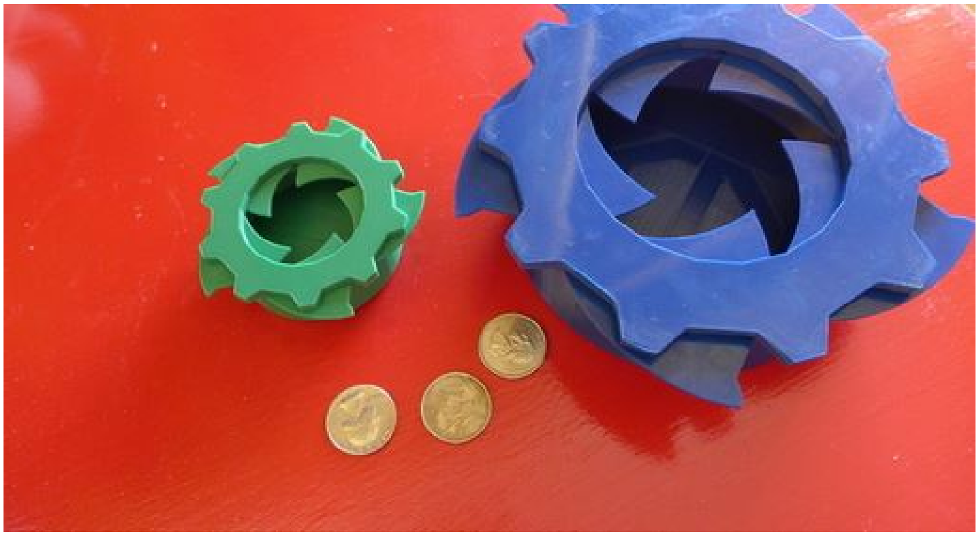

UP.257. ***If*** $A \in M_6(\mathbb{R})$ **such that**

$$det(A^4 + pA^2 + p^2 I_6) = det(A^2 + qI_6) = 0, p, q \in \mathbb{R}$$ **then find:**

$$\Omega = det(A)$$

Proposed by Marian Ursărescu-Romania

Solution by proposer

Let: $f(x) = x^4 + px^2 + p^2; g(x) = x^2 + q; f, g \in \mathbb{R}[x]$

P_A *–the characteristic polynomial of matrix* A*.*

We must show that: $(f, P_A) \neq 1$.

Suppose that: $(f, P_A) = 1 \Rightarrow \exists u, v \in \mathbb{R}[x]$ *such that:* $f(x)u(x) + P_A(x)v(x) = 1 \Rightarrow$
$f(A)u(A) + P_A(A)v(A) = I_6 \Rightarrow I_6 = O_6$ *contradiction!*

Similary $(f, g) \neq 1 \Rightarrow f / P_A$ *and* g / P_A*, for degree* $P_A = 6$

But $(f, g) = 1 \Rightarrow P_A(x) = f(x) \cdot g(x) \Rightarrow det(A) = P_A(0) = f(0) \cdot g(0) = p^2 q$

Note by Editor:

Many thanks to Florică Anastase-Romania for typed solution.