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If $x_1, x_2, \dots, x_n > 0$ then:

$$2\sum_{i=1}^{n}\left(\frac{1}{x_i + x_{i+1}}\prod_{\substack{j=1\\ j\neq i}}^{n} x_j\right) \le \prod_{\substack{j=1\\ j\neq i}}^{n} x_j,\quad x_{n+1} = x_1$$

Proposed by Marin Chirciu-Romania

Solution by Tran Hong-Dong Thap-Vietnam

$$2\sum_{i=1}^{n}\left(\frac{1}{x_i + x_{i+1}}\prod_{\substack{j=1\\ j\neq i}}^{n} x_j\right) = 2\left(\frac{x_2x_3\dots x_n}{x_1+x_2} + \frac{x_1x_3\dots x_n}{x_2+x_3} + \dots + \frac{x_1x_2\dots x_{n-1}}{x_n+x_1}\right)$$

$$= 2x_1x_2\dots x_n\left(\frac{1}{x_1(x_1+x_2)} + \frac{1}{x_2(x_2+x_3)} + \frac{1}{x_n(x_n+x_1)}\right) = \Omega$$

$$\frac{2}{2x_1(x_1+x_2)} \overset{CBS}{\leq} 2\cdot\frac{1}{4}\cdot\left(\frac{1}{2x_1} + \frac{1}{x_1+x_2}\right) \le \frac{2}{4}\cdot\left(\frac{1}{2x_1} + \frac{1}{4}\left(\frac{1}{x_1+x_2}\right)\right) = 2\left(\frac{3}{16x_1} + \frac{1}{16x_2}\right)$$

and analogs.

$$\Rightarrow \Omega \le 4x_1x_2\dots x_n\left[\frac{1}{16}\left(\left(\frac{3}{x_1}+\frac{1}{x_2}\right)+\left(\frac{3}{x_2}+\frac{1}{x_3}\right)+\cdots+\left(\frac{3}{x_n}+\frac{1}{x_1}\right)\right)\right]$$

$$=4x_1x_2\dots x_n\left(\frac{1}{16}\sum_{i=1}^{n}\frac{4}{x_i}\right)=4\cdot\frac{4}{16}x_1x_2\dots x_n\sum_{i=1}^{n}\frac{1}{x_i}=\sum_{i=1}^{n}\left(\prod_{\substack{j=1\\ i\ne j}}^{n}x_i\right)$$

Note by editor:

Mant thanks to Florică Anastase-Romania for typed solution.