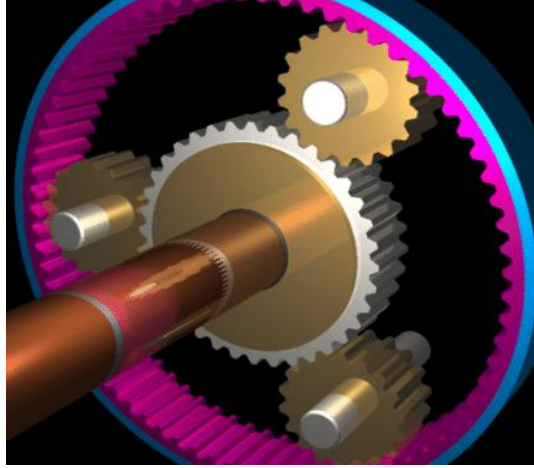


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If $x, y, z, t > 0$ then:

$$\frac{(xz - yt)^2 + (xz - yt)(xt + yz + yt) + (xt + yz + yt)^2}{xyzt} \geq 9$$

Proposed by Daniel Sitaru-Romania

Solution 1 by George Florin Șerban-Romania; Solution 2 by Sudir Jha-Kolkata-India; Solution 3 by Marian Voinea-Romania; Solution 4 by Myagmarsuren Yadamsuren-Darkhan-Mongolia

Solution 1 by George Florin Șerban-Romania

$$\begin{aligned} & \frac{x^2z^2 - 2xyzt + y^2t^2 + x^2zt + xyz^2 + xyzt - xyt^2 - y^2zt - y^2t^2}{xyzt} + \\ & \frac{x^2t^2 + y^2z^2 + y^2t^2 + 2xyzt + 2y^2zt + 2xyt^2}{xyzt} \geq 9 \\ & \frac{x^2z^2 + y^2z^2 + y^2t^2 + x^2t^2 + x^2zt + xyz^2 + xyzt + xyt^2 + y^2zt}{xyzt} \geq 9 \\ & \left(\frac{xz}{yt} + \frac{yt}{xz}\right) + \left(\frac{yz}{xt} + \frac{xt}{yz}\right) + \left(\frac{x}{y} + \frac{y}{x}\right) + \left(\frac{z}{t} + \frac{t}{z}\right) \geq 8 \\ & \left(\frac{xz}{yt} + \frac{yt}{xz}\right) + \left(\frac{yz}{xt} + \frac{xt}{yz}\right) + \left(\frac{x}{y} + \frac{y}{x}\right) + \left(\frac{z}{t} + \frac{t}{z}\right) \stackrel{Am-Gm}{\geq} \\ & 2\sqrt{\frac{xz}{yt} \cdot \frac{yt}{xz}} + 2\sqrt{\frac{yz}{xt} \cdot \frac{xt}{yz}} + 2\sqrt{\frac{x}{y} \cdot \frac{y}{x}} + 2\sqrt{\frac{z}{t} \cdot \frac{t}{z}} = 8 \text{ true.} \end{aligned}$$

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Solution 2 by Sudir Jha-Kolkata-India

$$\begin{aligned} LHS &= \frac{(xz - yt)^2 + (xz - yt)(xt + yz + yt) + (xt + yz + yt)^2}{xyzt} \\ &\stackrel{Am-Gm}{\geq} \frac{(xz - yt)^2 + 3\sqrt[3]{xy^2zt^2} \cdot 3\sqrt[3]{x^2yz^2t}}{xyzt} \\ &\geq \frac{0 + 9xyzt}{xyzt} = 9 \quad \text{Proved.} \end{aligned}$$

Solution 3 by Marian Voinea-Romania

$$\begin{aligned} \frac{(xz - yt)^2 + (xz - yt)(xt + yz + yt) + (xt + yz + yt)^2}{xyzt} &\geq 9 \Leftrightarrow \\ (xz - yt)^2 + (xz - yt)(xt + yz + yt) + (xt + yz + yt)^2 &\geq 9xyzt \Leftrightarrow \\ (xz - yt)(xz - yt + xt + yz + yt) + (xt + yz + yt)^2 &\geq 9xyzt \Leftrightarrow \\ (x^2z^2 + x^2t^2 + y^2z^2 + y^2t^2) + (x^2zt + xyz^2 + xyt^2 + y^2zt) &\stackrel{?}{\geq} 8xyzt \\ x^2z^2 + x^2t^2 + y^2z^2 + y^2t^2 &\stackrel{Am-Gm}{\geq} 4\sqrt[4]{x^4y^4z^4t^4} = 4xyzt \\ x^2zt + xyz^2 + xyt^2 + y^2zt &\stackrel{Am-Gm}{\geq} 4\sqrt[4]{x^4y^4z^4t^4} = 4xyzt \\ \text{Equality for } x = y = z = t & \end{aligned}$$

Solution 4 by Myagmarsuren Yadamsuren-Darkhan-Mongolia

$$\begin{aligned} \frac{(xz - yt)^2 + (xz - yt)(xt + yz + yt) + (xt + yz + yt)^2}{xyzt} &= \\ \frac{x^2z^2 + y^2t^2 - 2xyzt + (xt + yz + yt)(xz - yt + xt + yz + yt)}{xyzt} &\stackrel{Am-Gm}{\geq} \\ \frac{2xyzt - 2xyzt + 3\sqrt[3]{xy^2zt^2} \cdot 3\sqrt[3]{x^2yz^2t}}{xyzt} &= 9 \\ \text{Equality for } x = y = z = t & \end{aligned}$$

Note by Editor:

Many thanks to Florică Anastase-Romania for typed solutions.