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Solve for real numbers:

$$3\sqrt[3]{e^{3x} - e^x} - 2\sqrt{e^{2x} - e^x} = e^x + 1$$

Proposed by Jalil Hajimir-Toronto-Canada

Solution by Daniel Sitaru-Romania

By Rado's inequality:

$$3\left(\frac{a+b+c}{3} - \sqrt[3]{abc}\right) \geq 2\left(\frac{a+b}{2} - \sqrt{ab}\right)$$

$$3\sqrt[3]{abc} - 2\sqrt{ab} - c \leq 0, \text{ equality for } a = b = c$$

$$a = e^x, b = e^x - 1, c = e^x + 1$$

$$3\sqrt[3]{e^{3x} - e^x} - 2\sqrt{e^{2x} - e^x} - (e^x + 1) = 0 \Leftrightarrow$$

$$\Leftrightarrow 3\sqrt[3]{e^x(e^x - 1)(e^x + 1)} - 2\sqrt{e^x(e^x - 1)} - (e^x + 1) = 0 \Leftrightarrow$$

$$\Leftrightarrow e^x = e^x - 1 = e^x + 1. \text{ No solutions.}$$