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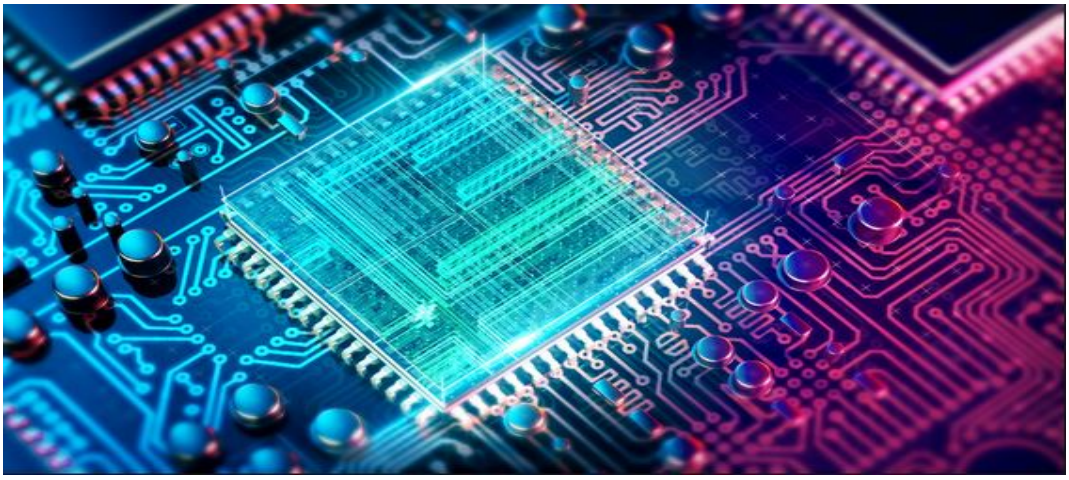

If $a, b \in \mathbb{R}, a \leq b, f: \mathbb{R} \to \left(0, \frac{\pi}{2}\right), f -$ continuous then:

$$4\int_a^b \csc(2f(x))\,dx + \int_a^b \cos\left(\frac{\pi}{4} - f(x)\right)dx \geq 5(b-a)$$

Proposed by Daniel Sitaru – Romania

Solution by Khaled Abd Imouti-Damascus-Syria

$$\int_a^b (4\csc(2f(x)) + \cos\left(\frac{\pi}{4} - f(x)\right)dx \overset{?}{\geq} 5(b-a)$$

Let be the function:

$$g(x) = 4\csc(2x) + \cos\left(\frac{\pi}{4} - x\right), x \in \left]0, \frac{\pi}{2}\right[$$

$$g(x) = \frac{4}{\sin 2x} + \cos\left(\frac{\pi}{4} - x\right), x \in \left]0, \frac{\pi}{2}\right[$$

$$\lim_{x \overset{>}{\to} 0}[g(x)] = +\infty, \lim_{x \overset{<}{\to} \frac{\pi}{2}}[g(x)] = +\infty$$

$$g'(x) = \frac{-8\cos 2x}{\sin^2 2x} + \sin\left(\frac{\pi}{4} - x\right)$$

$$g'(x) = 0 \Rightarrow x = \frac{\pi}{4}, g\left(\frac{\pi}{4}\right) = 5$$

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x	0		$\frac{\pi}{4}$		$\frac{\pi}{2}$
$g'(x)$	$-\ -\ -\ -\ -\ -\ -\ -\ -$		0	$+\ +\ +\ +\ +\ +\ +\ +\ +$	
$g(x)$	$+\infty$	$\searrow$	5	$\nearrow$	$+\infty$

So: $\int_a^b \left(4\csc\left(2f(x)\right)\right) + \cos\left(\frac{\pi}{4} - f(x)\right) dx \geq \int_a^b 5\,dx = 5(b-a)$