

# R M M

ROMANIAN MATHEMATICAL MAGAZINE  
www.ssmrmh.ro



**Prove without softs:**

$$\int_0^1 x^x(1-x)^x dx < \frac{3}{4}$$

*Proposed by Jalil Hajimir-Toronto-Canada*

*Solution by Daniel Sitaru-Romania*

$$x \in [0, 1] \Rightarrow x \geq 0, 1 - x \geq 0$$

$$x(1-x) \stackrel{AM-GM}{\leq} \left(\frac{x+1-x}{2}\right)^2 = \frac{1}{4}$$

$$(x(1-x))^x \leq \left(\frac{1}{4}\right)^x$$

$$\int_0^1 x^x(1-x)^x dx < \int_0^1 \left(\frac{1}{4}\right)^x dx = \frac{1}{\log \frac{1}{4}} \left( \left(\frac{1}{4}\right)^1 - \left(\frac{1}{4}\right)^0 \right)$$

R M M

ROMANIAN MATHEMATICAL MAGAZINE  
[www.ssmrmh.ro](http://www.ssmrmh.ro)

$$\int_0^1 x^x(1-x)^x dx < \frac{1}{-\log 4} \left( \frac{1}{4} - 1 \right) = \frac{3}{4 \log 4} < \frac{3}{4}$$