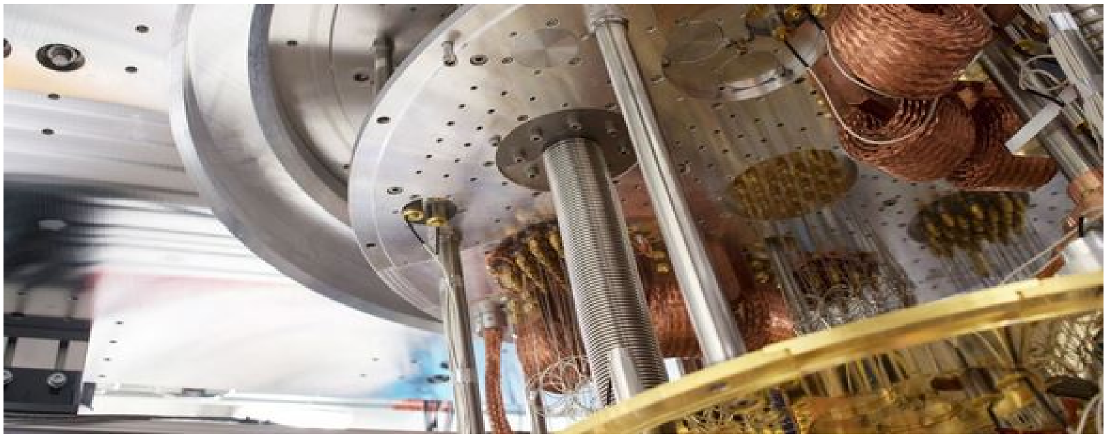

$$\Omega_1(t)=\int_0^1\left(\frac{x^{\sin^2 t}}{1+x^{\cos^2 t}}\right)dt,\ \Omega_2(t)=\int_0^1\left(\frac{x^{\cos^2 t}}{1+x^{\sin^2 t}}\right)dt$$

Prove that:

$$\Omega_1(t)+\Omega_2(t)\geq\frac{4-\sin 2t}{8+\sin 2t},\ t\in\left[0,\frac{\pi}{2}\right]$$

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Solution by Șerban George Florin – Romania

$$\frac{x^{\sin^2 t}}{1+x^{\cos^2 t}}+\frac{x^{\cos^2 t}}{1+x^{\sin^2 t}}\geq 2\sqrt{\frac{x^{\sin^2 t}+\cos^2 t}{\left(1+x^{\cos^2 t}\right)\left(1+x^{\sin^2 t}\right)}}=$$

$$=2\sqrt{\frac{x}{\left(1+x^{\cos^2 t}\right)\left(1+x^{\sin^2 t}\right)}}\geq 2\sqrt{\frac{x}{(1+1)(1+1)}}=\frac{2\sqrt{x}}{2}=\sqrt{x}$$

because: $x^{\cos^2 t}\leq 1=x^0,(\forall)x\in[0,1],0\leq\cos^2 t\leq 1$ ***and***

$$x^{\sin^2 t}\leq 1=x^0,(\forall)x\in[0,1],0\leq\sin^2 t\leq 1$$

$$\Rightarrow\Omega_1(t)+\Omega_2(t)=\int_0^1\left(\frac{x^{\sin^2 t}}{1+x^{\cos^2 t}}+\frac{x^{\cos^2 t}}{1+x^{\sin^2 t}}\right)dt\geq\int_0^1\sqrt{t}\,dt=$$

$$=\int_0^1 t^{\frac{1}{2}}\,dt=\left.\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right|_0^1=\left.\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^1=\frac{2}{3}\geq\frac{4-\sin 2t}{8+\sin 2t}$$

$\sin 2t = y \geq 0, t \in \left[0, \frac{\pi}{2}\right] \Rightarrow 2t \in [0, \pi] \Rightarrow 8 + y > 0$ ***and***

$4 - y > 0$ ***because*** $y \in [0, 1]$

$\Rightarrow 2(8 + y) \geq 3 \cdot (4 - y), 16 + 2y \geq 12 - 3y$

$\Rightarrow 4 + 5y \geq 0$***, true, because*** $y \geq 0$