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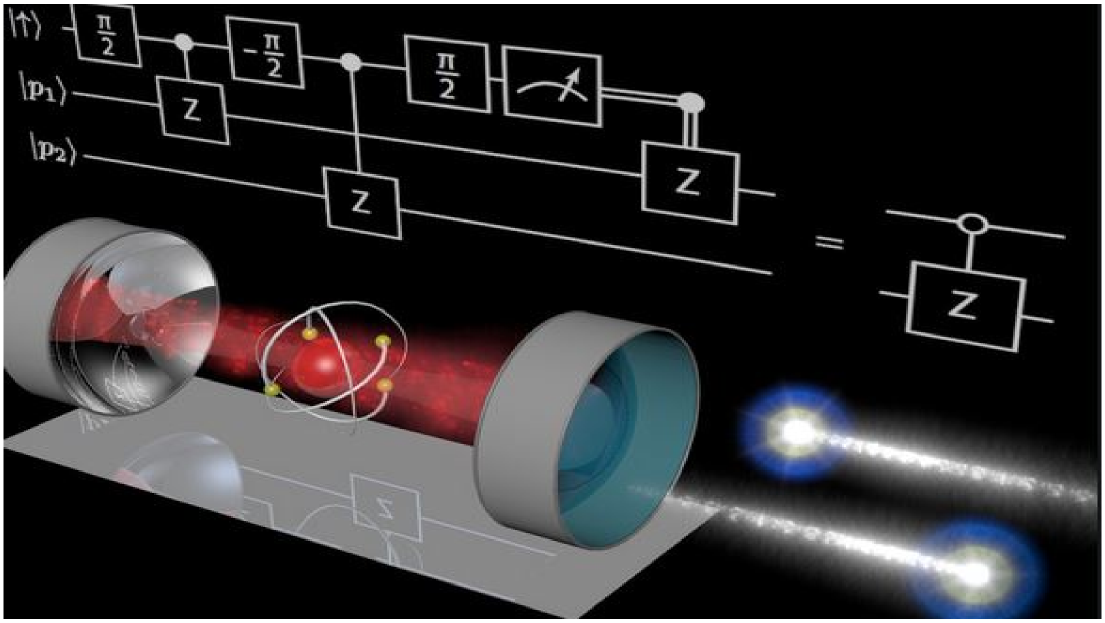

If $a,b,c\in(0,1)$ or $a,b,c\in(1,\infty)$, $a+b+c=abc$ then:

$$a^{a^2}\cdot b^{b^2}\cdot c^{c^2}\cdot(a+b+c)^{a^2+b^2+c^2}\geq(a^2+b^2+c^2)^{a^2+b^2+c^2}$$

Proposed by Florică Anastase-Romania

Solution by Adrian Popa-Romania

$$a,b,c\in(0,1)\ \textit{or}\ a,b,c\in(1,+\infty)$$

$$a^{a^2}\cdot b^{b^2}\cdot c^{c^2}\cdot(a+b+c)^{a^2+b^2+c^2}\geq(a^2+b^2+c^2)^{a^2+b^2+c^2}\,|\ln$$

$$a^2\ln a+b^2\ln b+c^2\ln c+(a^2+b^2+c^2)\ln(a+b+c)\overset{?}{\geq}(a^2+b^2+c^2)\ln(a^2+b^2+c^2)\Leftrightarrow$$

$$\Leftrightarrow a^2\ln a+b^2\ln b+c^2\ln c\geq(a^2+b^2+c^2)(\ln(a^2+b^2+c^2)-\ln(a+b+c))\Leftrightarrow$$

$$\Leftrightarrow a^2\ln a+b^2\ln b+c^2\ln c\geq(a^2+b^2+c^2)\ln\frac{a^2+b^2+c^2}{a+b+c}$$

Let be the function $f(x)=x\ln x\Rightarrow f'(x)=\ln x+1\Rightarrow f''(x)=\frac{1}{x}>0\Rightarrow f$ ***convexe***

Applying Jensen's inequality to function f: $\frac{af(a)+bf(b)+cf(c)}{a+b+c}\geq f\left(\frac{a^2+b^2+c^2}{a+b+c}\right)$

$\therefore \sum\lambda_i f(x_i)\geq f(\sum\lambda_i x_i)$. ***In our case:*** $\lambda_1=\frac{a}{a+b+c},\lambda_2=\frac{b}{a+b+c},\lambda_3=\frac{c}{a+b+c}\therefore$

$$\frac{a\cdot a\ln a+b\cdot b\ln b+c\cdot c\ln c}{a+b+c}\geq\frac{a^2+b^2+c^2}{a+b+c}\ln\frac{a^2+b^2+c^2}{a+b+c}\Bigg|\cdot(a+b+c)\Rightarrow$$

$\rightarrow a^2 \ln a + b^2 \ln b + c \ln c \geq (a^2 + b^2 + c^2) \cdot \ln \frac{a^2+b^2+c^2}{a+b+c}$ ***(true)***