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Find the last 3 digits of:

$$\Omega = 2019^{\overbrace{201920192019\ldots201953}^{}_{50\ times\ "2019"}}$$

Proposed by Naren Bhandari-Bajura-Nepal

Solution by Ajao Yinka-Nigeria

$$\underbrace{201920192019\ldots2019}_{2019\ \text{times}}53 =$$

$$= 2 \times 10^{201} + 0 \times 10^{200} + 1 \times 10^{199} + 9 \times 10^{198} + \cdots +$$

$$+2 \times 10^{5} + 0 \times 10^{4} + 1 \times 10^{3} + 9 \times 10^{2} + 5 \times 10 + 3 \times 10^{\circ}$$

We can use Euler's quotient function and Euler's theorem:

Since $1000 = 8 \times 125$

We evaluate $\Omega(mod\ 1000)$

Evaluating $\Omega(mod\ 8)$

$$\phi(8) = 4$$

$$2019 = 3(mod\ 8)$$

$$20192019\ldots201953 = 53 = 1(mod\ 4)$$

Since all other terms are multiples of 4.

So, $\Omega = 3' = 3(mod\ 8)$

Evaluating $\Omega(mod\ 125)$

$\phi(125) = 4 \times 25 = 100$

$2019 = 19\ mod\ 125$

$20192019\ldots201953 = 53\ (mod\ 100)$

Since all other terms are multiples of 100*, so* $\Omega = 19^{53}(mod\ 125)$

$= 19^{52} \times 19 = (19^4)^{13} \times 19 = 11^{13} \times 19$

$= (71^4)^3 \times 71 \times 19 = 56^3 \times 71 \times 19$

$= 116 \times 71 \times 19 = 109(mod\ 125)$

So, $\Omega = 3\ mod\ 8 = 3 + 8k$ *and* $\Omega = 109\ mod\ 125$

$3 + 8k = 109$ *mod* 125

$k = 107$ *mod* 125

So, $\Omega = 859\ mod\ 1000$

The last three digits are 859*.*