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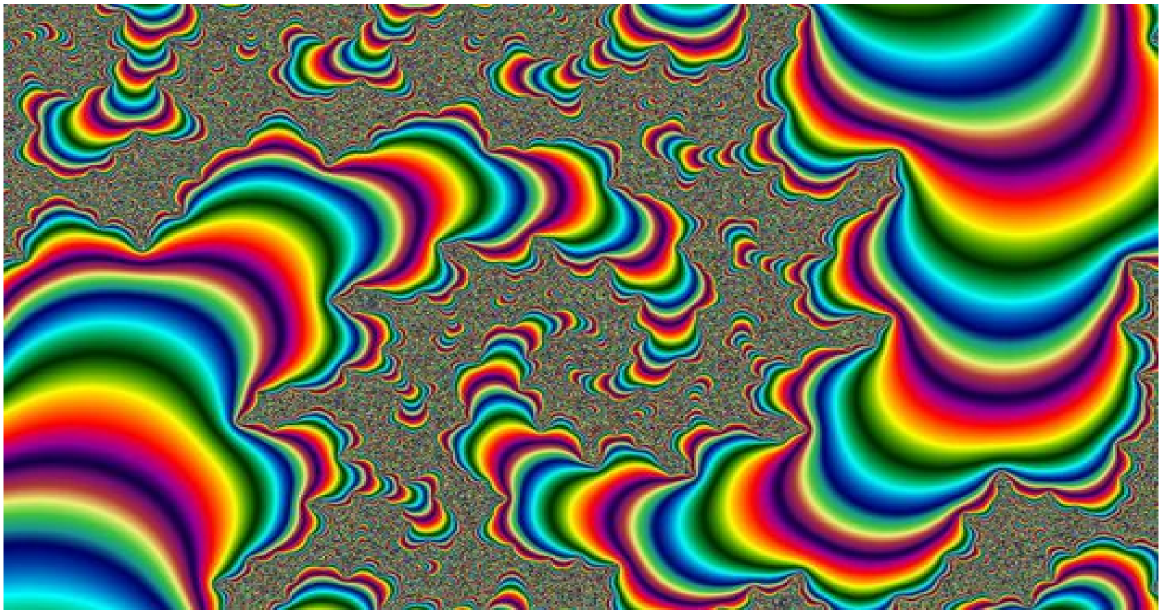
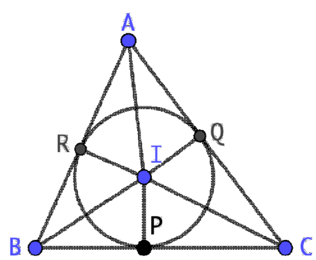


If in $\Delta ABC, I$ – incenter then:

$$[AIB]\cdot[AIC]+[BIC]\cdot[BIA]+[CIA]\cdot[CIB]\le r^2(R+r)^2$$

Proposed by Marian Ursărescu – Romania

Solution by Avishek Mitra-West Bengal-India



In $\Delta ABC \Rightarrow IP = IQ = IR = r$

$AB = a, BC = b, AC = c$

$$[AIB] = [BIA] = \frac{1}{2}r_a$$

$$[BIC] = [CIB] = \frac{1}{2}r_b$$

$$[AIC] = [CIA] = \frac{1}{2}r_c$$

$$\therefore \Omega = [AIB]\cdot[AIC] + [BIC]\cdot[BIA] + [CIA]\cdot[CIB]$$

$$= \frac{1}{4}r^2(ab+bc+ca) = \frac{1}{4}r^2(s^2+r^2+4Rr)$$

Need to show $\Rightarrow \frac{1}{4}r^2(s^2+r^2+4Rr) \le r^2(R+r)^2$

$$\Rightarrow \frac{s^2}{4} + \frac{r^2}{4} + Rr \le R^2 + 2Rr + r^2 \Rightarrow \frac{s^2}{4} \le R^2 + Rr + \frac{3r^2}{4}$$

$\Rightarrow s^2 \le 4R^2 + 4Rr + 3r^2$ ***(* True Gerretsen's Inequality)***

(Proved)