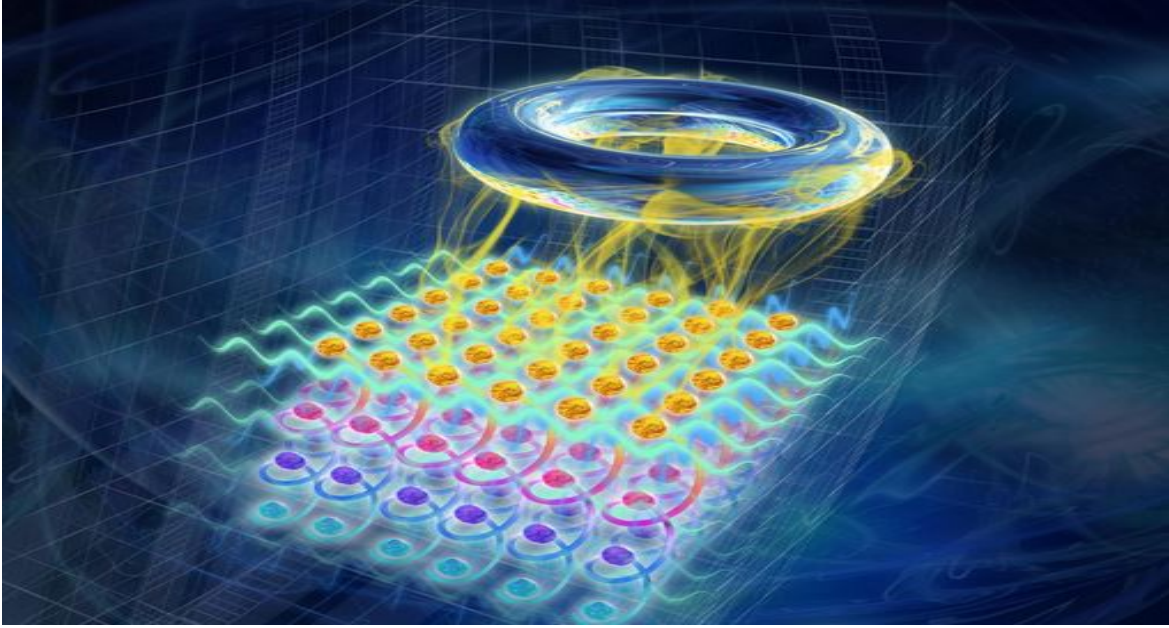


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If in ΔABC , $\frac{\pi}{2} > \mu(A), \mu(B), \mu(C) \geq \frac{\pi}{4}$ then:

$$\tan^2 A + \tan^2 B + \tan^2 C + \tan^2 A \cdot \tan^2 B \cdot \tan^2 C \geq 36$$

Proposed by Daniel Sitaru – Romania

Solution by Soumava Chakraborty-Kolkata-India

$$LHS \stackrel{(a)}{\geq} \frac{1}{3} \left(\sum \tan A \right)^2 + \pi \tan^2 A = \frac{(\pi \tan A)^2}{3} + (\pi \tan A)^2$$

$$= \frac{4x^2}{3} \quad (x = \pi \tan A) \stackrel{?}{\geq} 36 \Leftrightarrow x^2 \stackrel{?}{\geq} 27 \Leftrightarrow x \stackrel{?}{\geq} 3\sqrt{3}$$

$$\left(\because \tan A \tan B \tan C > 0 \text{ as } A, B, C < \frac{\pi}{2} \right)$$

$$\Leftrightarrow \sum \tan A \stackrel{?}{\underset{(1)}{\geq}} 3\sqrt{3}$$

$$\text{Let } f(x) = \tan x \quad \forall x \in \left[\frac{\pi}{4}, \frac{\pi}{2} \right)$$

$$f''(x) = 2 \tan x \sec^2 x > 0 \Rightarrow f(x) \text{ is convex}$$

$$\therefore \sum \tan A \stackrel{\text{Jensen}}{\geq} 3 \tan \left(\frac{A+B+C}{3} \right) = 3\sqrt{3}$$

$$\Rightarrow (1) \text{ is true} \Rightarrow \frac{4x^2}{3} \geq 36 \Rightarrow LHS \geq 36 \text{ (by (a)) (Proved)}$$