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ROMANIAN MATHEMATICAL MAGAZINE
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Let P be a point of circumcircle for equilateral ΔABC, P – is between B and C but different than B, C. Prove that:

$$\frac{1}{[PAB]}+\frac{1}{[PAC]}=\frac{1}{[PBC]}$$

Proposed by Ionuț Florin Voinea-Romania

Solution 1 by Marian Ursărescu-Romania, Solution 2 by Adrian Popa-Romania

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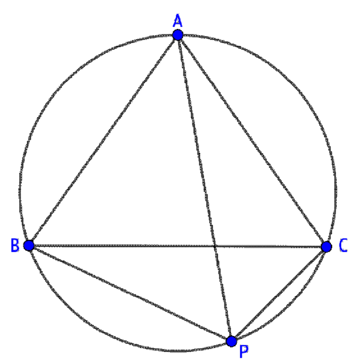
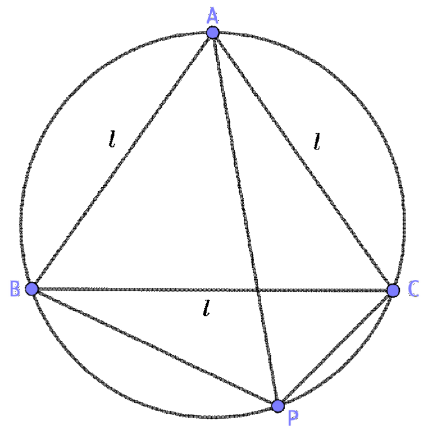


Because $S=\frac{abc}{4R}\Rightarrow$ *we must show:* $\frac{1}{PA\cdot PB\cdot AB}+\frac{1}{PA\cdot PC\cdot AC}=\frac{1}{PB\cdot PC\cdot BC}\Leftrightarrow$

$$\left.\begin{array}{c} PC\cdot AC\cdot BC+PB\cdot AB\cdot BC=PA\cdot AB\cdot AC \\ but\ \Delta ABC\ equilateral\ AB=BC=AC \end{array}\right\}\Rightarrow$$

we must show: $PB+PC=PA$*, but this relationship it is true because it is Schooten's theorem.*

Solution 2 by Adrian Popa-Romania



$$A_{ABP}=\frac{AB\cdot AP\cdot BP}{4R}=\frac{l\cdot AP\cdot BP}{4R}$$

$$A_{APC}=\frac{AC\cdot AP\cdot PC}{4R}=\frac{l\cdot AP\cdot PC}{4R}$$

$$A_{BPC}=\frac{BC\cdot BP\cdot PC}{4R}=\frac{l\cdot BP\cdot PC}{4R}$$

$$\frac{1}{A_{ABP}}+\frac{1}{A_{APC}}=\frac{1}{A_{BPC}}\Leftrightarrow\frac{4R}{l\cdot AP\cdot BP}+\frac{4R}{l\cdot AP\cdot PC}=\frac{4R}{l\cdot BP\cdot PC}\Big|\cdot\frac{l}{4R}\Leftrightarrow$$

$$\Leftrightarrow\frac{1}{AP\cdot BP}+\frac{1}{AP\cdot PC}=\frac{1}{BP\cdot PC}\Leftrightarrow\frac{PC+PB}{AP\cdot BP\cdot PC}=\frac{1}{BP\cdot PC}\Big|\cdot BP\cdot PC\Leftrightarrow$$

$$\Leftrightarrow\frac{PC+PB}{AP}=1\Leftrightarrow PC+PB=AP$$

In $ABPC$ – cyclic applying Ptolemy theorem: $AB\cdot PC+AC\cdot PB=AP\cdot BC$

$$l\cdot PC+l\cdot PB=AP\cdot l|:l\Rightarrow PC+PB=AP\Rightarrow\frac{1}{A_{ABP}}+\frac{1}{A_{APC}}=\frac{1}{A_{PBC}}$$