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# R M M

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DANIEL SITARU

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# PROBLEMS FOR JUNIORS

**JP.271.** If $a, b, c > 0; abc = a + b + c + 2$ then:

$$(2a+1)^2 + (2b+1)^2 + (2c+1)^2 \geq 75$$

*Proposed by Marin Chirciu - Romania*

**JP.272.** If $a, b, c, \lambda > 0; a^2 + b^2 + c^2 = 1$ then:

$$1 \leq a\sqrt{1+\lambda bc} + b\sqrt{1+\lambda ca} + c\sqrt{1+\lambda ab} \leq \sqrt{3+\lambda}$$

*Proposed by Hung Nguyen Viet - Vietnam*

**JP.273.** If $a, b, c > 0$ then:

$$\frac{a^3+b^3+c^3}{3abc} + \frac{ab+bc+ca}{a^2+b^2+c^2} \geq \frac{2(a^2+b^2+c^2)}{ab+bc+ca}$$

*Proposed by Hung Nguyen Viet - Vietnam*

**JP.274.** If $x, y, z \geq 0, x + y + z = 1; n \geq 2$ then:

$$(n+1)(xy+yz+zx) \leq n(x^2+y^2+z^2) + 9xyz$$

*Proposed by Marin Chirciu - Romania*

**JP.275.** If in $\Delta ABC, b^2 + c^2 = 3a^2$ then:

$$\frac{2}{h_a}\sqrt{\frac{bc}{5}} + \frac{w_b}{h_b} + \frac{w_c}{h_c} < 1 + \frac{r}{R}$$

*Proposed by Daniel Sitaru - Romania*

**JP.276.** In $\Delta ABC$ the following relationship holds:

$$\frac{3-n}{2} + \frac{nr}{R} \leq \frac{a^2}{b^2+c^2} + \frac{b^2}{c^2+a^2} + \frac{c^2}{a^2+b^2} \leq \frac{3R}{4r}; n \geq -1$$

*Proposed by Marin Chirciu - Romania*

**JP.277.** In $\Delta ABC$ the following relationship holds:

$$1 \leq \left(\frac{a}{m_b+m_c}\right)^2 + \left(\frac{b}{m_c+m_a}\right)^2 + \left(\frac{c}{m_a+m_b}\right)^2 \leq \frac{R}{2r}$$

*Proposed by Marin Chirciu - Romania*

**JP.278. Solve for real numbers ($a \geq 0$; fixed):**

$$\sqrt[3]{3x^2-3x+1}+4\sqrt[4]{4x^3-3x^4}=ax^5+(1-5a)x+4a+4$$

*Proposed by Marin Chirciu - Romania*

**JP.279. In $\Delta ABC$ the following relationship holds:**

$$\frac{1}{r_a(r_a+2r_b)}+\frac{1}{r_b(r_b+2r_c)}+\frac{1}{r_c(r_c+2r_a)}\leq\frac{1}{9r^2}$$

*Proposed by Hung Nguyen Viet - Vietnam*

**JP.280. In $\Delta ABC$ the following relationship holds:**

$$\sqrt[3]{r_a^4r_b^2}+\sqrt[3]{r_b^4r_c^2}+\sqrt[3]{r_c^4r_a^2}\leq\frac{(4R+r)^2}{3}$$

*Proposed by Hung Nguyen Viet - Vietnam*

**JP.281. If $a,b,c>0; abc=1$ then:**

$$\frac{(a+b)^2}{\sqrt{a^2+b^2}}+\frac{(b+c)^2}{\sqrt{b^2+c^2}}+\frac{(c+a)^2}{\sqrt{c^2+a^2}}\geq 6\sqrt{2}$$

*Proposed by Nguyen Viet Hung - Vietnam*

**JP.282. If $a,b,c>1$ then:**

$$\log a\cdot\log b\cdot\log c\cdot(\log_a e+\log_b e+\log_c e)^2\geq 3\log(abc)$$

*Proposed by Daniel Sitaru - Romania*

**JP.283. If $a,b,c\in\mathbb{R}$ then:**

$$2\sum_{cyc}\sin^2 a+\sum_{cyc}\sin^2(a+b)\leq\frac{27}{4}$$

*Proposed by Daniel Sitaru - Romania*

**JP.284. In acute $\Delta ABC$ the following relationship holds:**

$$\frac{\sqrt{\sin 2A}+\sqrt{\sin 2B}+\sqrt{\sin 2C}}{\sqrt{\tan A}+\sqrt{\tan B}+\sqrt{\tan C}}\geq\sqrt{2\left(\frac{r}{R}+1\right)^2-4}$$

*Proposed by Marian Ursărescu - Romania*

**JP.285. In $\Delta ABC$ the following relationship holds:**

$$\frac{m_a^2}{m_b}+\frac{m_b^2}{m_c}+\frac{m_c^2}{m_a}\geq s\sqrt{3}$$

*Proposed by Marian Ursărescu - Romania*

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# PROBLEMS FOR SENIORS

**SP.271.** If $a_1, a_2, \ldots, a_n > 0; a_1 a_2 \cdot \ldots \cdot a_n = 1; \lambda \geq \frac{1}{2}$ then:

$$\frac{1}{\lambda + a_1} + \frac{1}{\lambda + a_2} + \ldots + \frac{1}{\lambda + a_n} \leq \frac{a_1 + a_2 + \ldots + a_n}{\lambda + 1}$$

*Proposed by Marin Chirciu - Romania*

**SP.272.** In $\Delta ABC$ the following relationship holds:

$$\frac{3}{R} \leq \frac{r_a + r_b}{c^2} + \frac{r_b + r_c}{a^2} + \frac{r_c + r_a}{b^2} \leq \frac{3}{4r}\left(\frac{R^2}{r^2} - 2\right)$$

*Proposed by George Apostolopoulos - Greece*

**SP.273.** If $x, y \in \mathbb{R}$ then:

$$\sin^4 x + \cos^4 x \sin^4 y + \cos^4 x \cos^4 y \geq \frac{1}{3}$$

When does the equality holds?

*Proposed by Daniel Sitaru - Romania*

**SP.274.** If in $\Delta ABC; s = \frac{1}{2}$ then the following relationship holds:

$$a \cdot e^{\frac{m_a}{a}} + b \cdot e^{\frac{m_b}{b}} + c \cdot e^{\frac{m_c}{c}} \geq e^{m_a + m_b + m_c}$$

*Proposed by Daniel Sitaru - Romania*

**SP.275.** In $\Delta ABC$ the following relationship holds:

$$\left(\frac{a + b}{m_a + m_b}\right)^2 + \left(\frac{b + c}{m_b + m_c}\right)^2 + \left(\frac{c + a}{m_c + m_a}\right)^2 \geq 4$$

*Proposed by Hung Nguyen Viet - Vietnam*

**SP.276.** If $x, y, z > 0; n \geq 1$ then:

$$\sum_{cyc} \frac{(nx + y)(nx + z)}{yz} \geq \frac{(n + 1)^2}{2} \sum_{cyc} \frac{y + z}{x}$$

*Proposed by Marin Chirciu - Romania*

**SP.277.** In $\Delta ABC$ the following relationship holds:

$$27\left(\frac{R}{2r}\right)^2 - \sum_{cyc}\left(\sqrt{\frac{\sin A}{\sin B}} + \sqrt{\frac{\sin A}{\sin C}}\right)^3 \geq 3$$

*Proposed by George Apostolopoulos - Greece*

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**SP.278. Let be $f:\left[\frac{\pi}{4},\frac{3\pi}{4}\right]\to\mathbb{R}; f(x)=\frac{\cot^2 x-2\cot x+n-1}{\cot^2 x+2\cot x+n+1}; n\geq 2$. Find Imf.**

*Proposed by Marin Chirciu - Romania*

**SP.279. If in $\Delta ABC;\omega$ - Brocard angle then the following relationship holds:**

$$\frac{1}{2\sin\omega}\geq\sqrt{\frac{w_a w_b w_c}{h_a h_b h_c}}\geq\frac{2\cos\omega}{\sqrt{3}}$$

*Proposed by Vasile Jiglău - Romania*

**SP.280. If $x,y,z\geq 0;\{x\}^9+\{y\}^9+\{z\}^9=\frac{1}{64}$ then:**

$$x^7\cdot[x]\cdot\{x\}+y^7\cdot[y]\cdot\{y\}+z^7\cdot[z]\cdot\{z\}<64([x]^9+[y]^9+[z]^9)+1$$

**$\{x\}=x-[x];[*]$ - great integer function**

*Proposed by Daniel Sitaru - Romania*

**SP.281. If $x\in\left(0,\frac{\pi}{2}\right);a,b>0$ then:**

$$\left(\left(\sqrt{\frac{a}{b}}\right)^{\frac{\sin x}{x}}+\left(\sqrt{\frac{b}{a}}\right)^{\frac{\sin x}{x}}\right)\cdot\left(\left(\sqrt{\frac{a}{b}}\right)^{\frac{x}{\tan x}}+\left(\sqrt{\frac{b}{a}}\right)^{\frac{x}{\tan x}}\right)\leq\left(\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}\right)^2$$

*Proposed by Daniel Sitaru - Romania*

**SP.282. If in $\Delta ABC; H$ - orthocentre; $HD, HE, HF$ bisectors of angles $BHC, CHA$ respectively $AHB; D\in(BC); E\in(CA)$; $F\in(AB)$ then the following relationship holds:**

$$\frac{[DEF]}{[ABC]}\geq 13\left(\frac{r}{R}\right)^2-3$$

*Proposed by Marian Ursărescu - Romania*

**SP.283. Find $x,y>0$ such that:**

$$\sqrt{\frac{x}{y}}+\sqrt[3]{\frac{3}{x}}+\sqrt[5]{\frac{y}{3}}=\frac{10}{\sqrt[10]{337500}}$$

*Proposed by Daniel Sitaru - Romania*

**SP.284. In $\Delta ABC$ the following relationship holds:**

$$\frac{1}{3R^2}\leq\frac{1}{(r_a+r_b)^2}+\frac{1}{(r_b+r_c)^2}+\frac{1}{(r_c+r_a)^2}\leq\frac{R^2-3r^2}{12r^4}$$

*Proposed by George Apostolopoulos - Greece*


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**SP.285. In $\Delta ABC$ the following relationship holds:**

$$\frac{3r}{R} \leq \frac{h_a}{r_b + r_c} + \frac{h_b}{r_c + r_a} + \frac{h_c}{r_a + r_b} \leq \frac{3}{2}$$

*Proposed by George Apostolopoulos - Greece*

# UNDERGRADUATE PROBLEMS

**UP.271. If $0 < a \leq b < \frac{\pi}{4}$ then:**

$$\int_a^b \int_a^b \int_a^b \left( \cos\left(\frac{\pi}{4} - x\right) \cos\left(\frac{\pi}{4} - y\right) \cos\left(\frac{\pi}{4} - z\right)\right) dxdydz \geq$$

$$\geq \sin^3(b + a) \cdot \sin^3(b - a)$$

*Proposed by Daniel Sitaru - Romania*

**UP.272. Prove without softs:**

$$\int_0^{\frac{\pi}{4}} \int_0^{\frac{\pi}{4}} \int_0^{\frac{\pi}{4}} (\tan(\sqrt[3]{xyz}))^3 dxdydz < \frac{\log^3 2}{8}$$

*Proposed by Florentin Vişescu - Romania*

**UP.273. In acute $\Delta ABC$ the following relationship holds:**

$$\tan(\sqrt{AB}) + \tan(\sqrt{BC}) + \tan(\sqrt{CA}) \leq \tan A + \tan B + \tan C$$

*Proposed by Florentin Vişescu - Romania*

**UP.274. If $\omega = 1 - \frac{1}{3}\binom{n}{1} + \frac{1}{5}\binom{n}{2} + \ldots + \frac{(-1)^n}{2n+1}\binom{n}{n}$ then find:**

$$\Omega = \lim_{n\to\infty} \left(1 + \frac{\sqrt[n]{\omega_n}}{n!}\right)^{\frac{n!}{e^n}}$$

*Proposed by Florică Anastase - Romania*

**UP.275. Find:**

$$\Omega = \lim_{n\to\infty} \left( \frac{1}{n^8} \sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{j} \sum_{l=1}^{k} (ijkl) \right)$$

*Proposed by Daniel Sitaru - Romania*

**UP.276. Find:**

$$\Omega = \lim_{n\to\infty}\left(\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j}\sum_{l=1}^{k}\left(\frac{1}{2^{i+j+k+l}}\right)\right)$$

*Proposed by Daniel Sitaru - Romania*

**UP.277. Find:**

$$\Omega = \lim_{n\to\infty}\left(\frac{\sum_{1\le i<j<k\le n}\left(\frac{1}{\sqrt[3]{(ijk)^2}}\right)}{e^{H_n}}\right)$$

*Proposed by Marian Ursărescu - Romania*

**UP.278. If $a, b \in \mathbb{R}$ then:**

$$\int_a^b\int_a^b(\cos x\cos y\cos(x+y))dxdy + \frac{1}{8}(b-a)^2 \ge 0$$

*Proposed by Daniel Sitaru - Romania*

**UP.279. If $a > 0; f : (0,\infty) \to (0,\infty); \lim_{x\to\infty}\left(\frac{f(x+1)\cdot x^a}{f(x)}\right) = b > 0$ and exists $\lim_{x\to\infty}((f(x))^{\frac{1}{x}}\cdot x^a)$ then find:**

$$\Omega = \lim_{x\to\infty}\left(\left(\left(\Gamma(x+2)\right)^{\frac{a}{x+1}} - \left(\Gamma(x+1)\right)^{\frac{a}{x}}\right)^{x\cdot(f(x))^{\frac{1}{x}}}\right)$$

*Proposed by D.M. Bătineţu - Giurgiu, Neculai Stanciu - Romania*

**UP.280. Let $a, b, c$ be sides in $\Delta ABC; (x_n)_{n\ge1}; (y_n)_{n\ge1}; (z_n)_{n\ge1}$ sequences of positive numbers such that:**

$$\lim_{n\to\infty}(x_{n+1}-x_n) = a; \lim_{n\to\infty}\left(\frac{y_{n+1}}{ny_n}\right) = b; \lim_{n\to\infty}\left(\frac{z_{n+1}}{nz_n}\right) = c$$

**Prove that:**

$$\lim_{n\to\infty}\left(\frac{x_n\cdot\sqrt[n]{y_n} + e\cdot\sqrt[n]{y_nz_n} + x_n\cdot\sqrt[n]{z_n}}{n^2}\right) \ge \frac{4\sqrt{3}F}{e}$$

*Proposed by D.M. Bătineţu - Giurgiu, Neculai Stanciu - Romania*

**UP.281. If $(a_n)_{n\ge1} \subset (0,\infty); \lim_{n\to\infty}\left(\frac{a_{n+1}}{n^2\cdot a_n}\right) = a > 0;$**
**$x_1 = a_1; x_2 = a_1\sqrt{a_2}; x_3 = a_1\cdot\sqrt{a_2}\cdot\sqrt[3]{a_3}, \cdots,$**
**$x_n = a_1\sqrt{a_2}\sqrt[3]{a_3}\cdot\ldots\cdot\sqrt[n]{a_n}$ then find:**

$$\Omega = \lim_{n\to\infty}\left(\frac{(n+1)^3}{\sqrt[n+1]{x_{n+1}}} - \frac{n^3}{\sqrt[n]{x_n}}\right)$$

*Proposed by D.M. Bătineţu - Giurgiu, Daniel Sitaru - Romania*


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**UP.282. If $m, p, r, s, t \geq 0; (a_n)_{n\geq 1}; (b_n)_{n\geq 1}; (c_n)_{n\geq 1} \subset (0, \infty)$;**

$$\lim_{n\to\infty}\left(\frac{a_{n+1}}{a_n \cdot n^r}\right) = a > 0; \lim_{n\to\infty}\left(\frac{b_{n+1}}{b_n \cdot n^s}\right) = b > 0;$$

$$\lim_{n\to\infty}\left(\frac{c_{n+1} - c_n}{n^t}\right) = c > 0 \text{ then:}$$

$$\lim_{n\to\infty}\left(\frac{c_{n+1} \cdot \sqrt[n+1]{a_{n+1}^m \cdot b_{n+1}^p}}{(n+1)^{mr+ps+t}} - \frac{c_n \cdot \sqrt[n]{a_n^m \cdot b_n^p}}{n^{mr+ps+t}}\right) = \frac{a^m \cdot b^p \cdot c}{(t+1) \cdot e^{mr+ps}}$$

*Proposed by D.M. Bătineţu - Giurgiu, Neculai Stanciu - Romania*

**UP.283. In $\Delta ABC$ the following relationship holds:**

$$\frac{r}{4R^4} \leq \frac{h_a}{a^2(b+c)^2} + \frac{h_b}{b^2(c+a)^2} + \frac{h_c}{c^2(a+b)^2} \leq \frac{1}{64r^3}$$

*Proposed by George Apostolopoulos - Greece*

**UP.284. In $\Delta ABC$ the following relationship holds:**

$$\frac{F}{12R^2(R-r)} \leq \sum_{cyc} \frac{ab}{(2a^2+b^2+c^2)(b+c)} \leq \frac{\sqrt{3}}{16r}$$

*Proposed by George Apostolopoulos - Greece*

**UP.285. In $\Delta ABC$ the following relationship holds:**

$$\left(\frac{15}{2} - \frac{3R^2}{4r^2}\right)R \leq \frac{w_a^2}{h_a} + \frac{w_b^2}{h_b} + \frac{w_c^2}{h_c} \leq \frac{9R}{2}$$

*Proposed by George Apostolopoulos - Greece*

Mathematics Department, "Theodor Costescu" National Economic, College Drobeta Turnu - Severin, ROMANIA