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UP.251. If $t,u,x,y>0$ then in ΔABC the following relationship holds:

$$\sum_{cyc}\frac{a^2}{\left(t\cos^2\frac{A}{2}+u\cos^2\frac{B}{2}\right)(xb+yc)^2}\geq\frac{18R}{(t+u)(x+y)^2(4R+r)}$$

Proposed by D.M. Bătinețu – Giurgiu, Neculai Stanciu – Romania

Solution by Marian Ursărescu – Romania

$$\frac{\sum\left(\frac{a}{xb+yc}\right)^2}{t\cos^2\frac{A}{2}+y\cos^2\frac{B}{2}}\overset{Bergstrom}{\geq}\frac{\left(\sum\frac{a}{xb+yc}\right)^2}{(t+y)\cdot\sum\cos^2\frac{A}{2}}\quad (1)$$

But $\sum\cos^2\frac{A}{2}=\frac{4R+r}{2R}$ *(2)*

From (1)+(2) we must show: $\frac{\left(\sum\frac{a}{xb+yc}\right)\cdot 2R}{(t+y)(4R+r)}\geq\frac{18R}{(t+y)(x+y)^2(4R+r)}\Leftrightarrow$

$$\left(\sum\frac{a}{xb+yc}\right)^2\geq\frac{9}{(x+y)^2}\Leftrightarrow\sum\frac{a}{xb+yc}\geq\frac{3}{x+y}\quad (3)$$

But $\sum\frac{a}{xb+yc}=\sum\frac{a^2}{abx+acy}\overset{Bergstrom}{\geq}\frac{(a+b+c)^2}{(x+y)(ab+ac+bc)}$ *(4)*

But $(a+b+c)^2\geq 3(ab+ac+bc)$ *(5)*

From (4)+(5)$\Rightarrow\sum\frac{a}{xb+yc}\geq\frac{3}{x+y}\Rightarrow$ *(3) it is true.*