PROPOSED PROBLEM

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Let a, b and c be complex numbers such that $abc = 1$. Find the value of the cubic root of:

$$\begin{vmatrix} b+n^3c & n(c-b) & n^2(b-c) \\ n^2(c-a) & c+n^3a & n(a-c) \\ n(b-a) & n^2(a-b) & a+n^3b \end{vmatrix}$$

Solution 1 by Adrian Popa - Romania.

$$\Delta = \begin{vmatrix} b+n^3c & nc-nb & n^2b-n^2c \\ n^2c-n^2a & c+n^3a & na-nc \\ nb-na & n^2a-n^2b & a+n^3b \end{vmatrix}$$

$$(b+n^3c)(ac+n^3bc+n^3a^2+n^6ab-n^3a^2+n^3ac+n^3ab-n^3bc) =$$
$$= abc+n^6ab^2+n^3abc+n^3ab^2+n^2ac^2+n^9abc+n^6abc+n^6ac^2$$
$$= 1+n^3+n^6+n^9+n^6ab^2+n^3ab^2+n^3ac^2+n^6ac^2 \quad (1)$$

$$(nc-nb)(n^2ac+n^5bc-n^2a^2-n^5ab-n^2ab+n^2bc+n^2a^2-n^2ac) =$$
$$n^6bc^2-n^6abc-n^3abc+n^3bc^2-n^6b^2c+n^6ab^2+n^3ab^2-n^3b^2c =$$
$$= -n^6-n^3+n^6bc^2+n^3bc^2-n^6b^2c-n^3b^2c+n^6ab^2+n^3ab^2 \quad (2)$$

$$(n^2b-n^2c)(n^4ac-n^4bc-n^4a^2+n^4ab-nbc-n^4ab+nac+n^4a^2) =$$
$$= n^6abc-n^6b^2c-n^3b^2c+n^3abc-n^6ac^2+n^6bc^2+n^3bc^2-n^3ac^2 =$$
$$= n^6+n^3-n^6b^2c-n^3b^2c-n^6ac^2-n^3ac^2+n^6bc^2+n^3bc^2 \quad (3)$$

$$\Delta = (1)-(2)+(3)$$
$$\Delta = n^9+3n^6+3n^3+1$$
$$\Delta(n^3+1)^3 \Rightarrow \sqrt[3]{\Delta} = n^3+1$$

□

Solution 2 by Ravi Prakash - New Delhi - India.

Let $\Delta = \begin{vmatrix} b+n^3c & n(c-b) & n^2(b-c) \\ n^2(c-a) & c+n^3a & n(a-c) \\ n(b-a) & n^2(a-b) & a+n^2b \end{vmatrix}$

$$c_3 \to c_3+nc_2, c_2 \to c_2+nc_1$$

$$\Delta = \begin{vmatrix} b+n^3c & (n^3+1)nc & 0 \\ n^2(c-a) & (n^3+1)c & (n^3+1)na \\ n(b-a) & 0 & (n^3+1)a \end{vmatrix}$$

$$= (n^3+a)^2ac \begin{vmatrix} b+n^3c & n & 0 \\ n^2(c-a) & 1 & n \\ n(b-a) & 0 & 1 \end{vmatrix}$$

$$c_1 \to c_1-n^2cc_2+nac_3$$

$$\Delta = (n^3+1)^2 ac \begin{vmatrix} b & n & 0 \\ 0 & 1 & n \\ nb & 0 & 1 \end{vmatrix} = (n^3+1)^2 acb \begin{vmatrix} 1 & n & 0 \\ 0 & 1 & n \\ n & 0 & 1 \end{vmatrix}$$
$$= (n^3+1)^3(1) = (n^3+1)^3 \Rightarrow \Delta^{\frac{1}{3}} = n^3+1$$

□

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