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In acute ΔABC the following relationship holds:

$$\cot A \cdot \cot B \cdot \cot C \geq \frac{(s - r - 2R)(s + r + 2R)}{Rs}$$

Proposed by Florică Anastase-Romania

Solution by Soumava Chakraborty-Kolkata-India

$$\begin{aligned} \prod \cot A &= \frac{\prod \cos A}{\prod \sin A} = \frac{\frac{s^2 - (2R + r)^2}{4R^2}}{\frac{abc}{8R^3}} \\ &= \frac{(s - r - 2R)(s + r + 2R)}{4R^2} \cdot \frac{8R^3}{4Rrs} = \frac{(s - r - 2R)(s + r + 2R)}{2rs} \\ &\stackrel{\text{Euler}}{\geq} \frac{(s - r - 2R)(s + r + 2R)}{Rs} \end{aligned}$$

$$\because (s - r - 2R)(s + r + 2R) > 0 \text{ as } \prod \cos A = \frac{s^2 - (2R + r)^2}{4R^2} > 0 \text{ as } \Delta ABC \text{ is acute}$$

Proved