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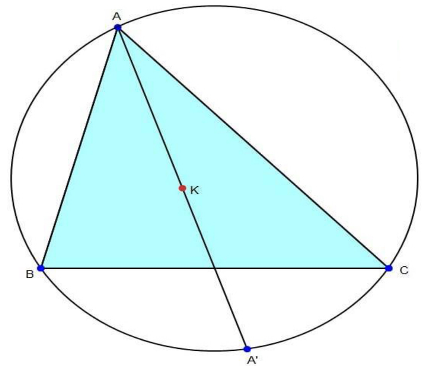

K – symmedian point of ABC, A' intersection point of AK with circumcircle

$$2a^2 = b^2 + c^2 \Rightarrow AK = KA'$$

Proposed by Abdilkadir Altintas-Afyonkarashisar-Turkey

Solution 1 by Marian Ursărescu-Romania, Solution 2 by Thanasis Gakopoulos-Larisa-Greece

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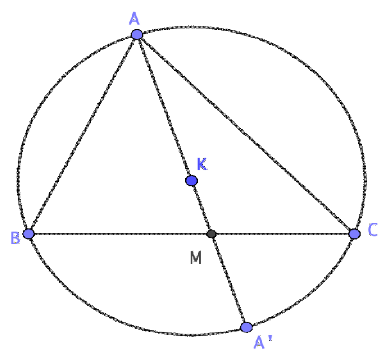


$$\rho(K) = -AK \cdot KA' = OK^2 - R^2 \Rightarrow AK \cdot KA' = R^2 - OK^2 \quad (1)$$

$$\text{But } R^2 - OK^2 = \frac{3a^2b^2c^2}{(a^2+b^2+c^2)^2} \quad (2)$$

$$(1)+(2) \Rightarrow AK \cdot KA' = \frac{3a^2b^2c^2}{(a^2+b^2+c^2)^2} \quad (3)$$

$$\text{But } \frac{AK}{KM} = \frac{b^2+c^2}{a^2} \Rightarrow \frac{AK}{S_a} = \frac{b^2+c^2}{a^2+b^2+c^2} \Rightarrow AK = \frac{b^2+c^2}{a^2+b^2+c^2} S_a =$$

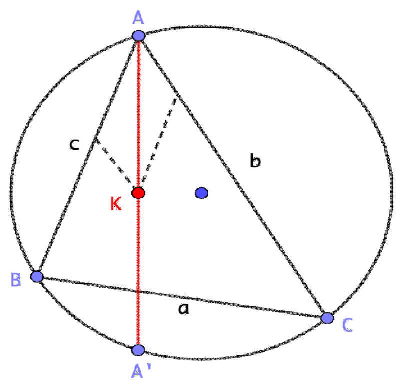
$$= \frac{b^2+c^2}{(a^2+b^2+c^2)} \cdot \frac{2bc}{(b^2+c^2)} \cdot m_a = \frac{2bc}{a^2+b^2+c^2} \cdot m_a \quad (4)$$

$$AK = KA' \overset{from\ (3)}{\Leftrightarrow} AK = \frac{3a^2b^2c^2}{(a^2+b^2+c^2)^2 AK} \Leftrightarrow$$

$$KA^2 = \frac{3a^2b^2c^2}{(a^2+b^2+c^2)^2} \Leftrightarrow \frac{4b^2c^2}{(a^2+b^2+c^2)^2} m_a^2 = \frac{3a^2b^2c^2}{(a^2+b^2+c^2)^2} \Leftrightarrow$$

$$\left.\begin{array}{r} t_1 m_a^2 = 3a^2 \Leftrightarrow 2(b^2+c^2) = a^2 = 3a^2 \\ But\ b^2+c^2 = 2a^2 \end{array}\right\} \Rightarrow 3a^2 = 3a^2 \textit{ true}$$

Solution 2 by Thanasis Gakopoulos-Larisa-Greece



K **– Lemoine's point,** $2a^2 = b^2 + c^2 \to AK = KA$

$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{b^2+c^2}{4bc} \quad (*)$$

PLAGIOGONAL system: $AB \equiv Ax, AC \equiv Ay$

$$\left.\begin{array}{l} A(0,0), K(K_1,K_2)\ K_1 = \dfrac{b^2c}{a^2+b^2+c^2} = \dfrac{2b^2c}{3(b^2+c^2)}, K_2 = \dfrac{bc^2}{a^2+b^2+c^2} = \dfrac{2bc^2}{3(b^2+c^2)} \\ \left.\begin{array}{r} AK: \dfrac{x}{K_1} = \dfrac{y}{k_2} \to cd = by \\ (c): x^2+y^2+2xy\cdot\cos A - cx - by = 0 \end{array}\right\} \overset{(x)}{\to} A'\left(\dfrac{4b^2c}{3(a^2+b^2)}, \dfrac{4bc^2}{3(a^2+b^2)}\right) \end{array}\right\}$$

$$\to AK = KA'$$