

R M M

ROMANIAN MATHEMATICAL MAGAZINE
www.ssmrmh.ro



$$\text{In } \Delta ABC, AB = \sqrt{2(\tan 20^\circ + \tan 33^\circ)}, b = \sqrt{2(\tan 33^\circ + \tan 37^\circ)}$$

$$c = \sqrt{2(\tan 37^\circ + \tan 20^\circ)}$$

Find: $\Omega = [ABC]$

Proposed by Adil Abdullayev-Baku-Azerbaijan

Solution by Avishek Mitra-West Bengal-India

$$\text{Let } \tan 20 = x, \tan 33 = y, \tan 37 = z$$

$$\text{In } \Delta ABC \Rightarrow a = \sqrt{2(x+y)}, b = \sqrt{2(y+z)}, c = \sqrt{2(z+x)}$$

$$s = \frac{a+b+c}{2}$$

$$\Leftrightarrow \Omega = [ABC] = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \frac{1}{4} \sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)} = \frac{1}{4} \sqrt{2 \sum a^2 b^2 - \sum a^4}$$

$$= \frac{1}{4} \sqrt{\sum 8(x+y)(y+z) - 4 \sum (x+y)^2}$$

$$= \frac{1}{4} \sqrt{24 \sum xy + 8 \sum x^2 - 8 \sum xy - 8 \sum x^2} = \frac{1}{4} \sqrt{16 \sum xy} = \sqrt{\sum xy}$$

$$= \sqrt{\tan 20 \cdot \tan 33 + \tan 33 \cdot \tan 37 + \tan 37 \cdot \tan 20}$$

R M M

ROMANIAN MATHEMATICAL MAGAZINE
www.ssmrmh.ro

$$\begin{aligned} &= \sqrt{\tan 33 \left(\frac{\sin 20 \cdot \cos 37 + \cos 20 \cdot \sin 37}{\cos 20 \cdot \cos 37} \right) + \tan 37 \cdot \tan 20} \\ &= \sqrt{\frac{\sin 33}{\cos 33} \cdot \frac{\sin 57}{\cos 20 \cdot \cos 37} + \tan 37 \cdot \tan 20} \\ &= \sqrt{\frac{\sin 33}{\cos 20 \cdot \cos 37} + \frac{\sin 37 \cdot \sin 20}{\cos 20 \cdot \cos 37}} \quad [\because \cos 33 = \sin 57] \\ &= \sqrt{\frac{\sin 33 + \frac{1}{2}(\cos 17 - \cos 57)}{\cos 20 \cdot \cos 37}} = \sqrt{\frac{\frac{1}{2}(\cos 17 + \cos 57)}{\frac{1}{2}(\cos 57 + \cos 17)}} = \mathbf{1} \quad (\text{Answer}) \end{aligned}$$