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Find without softs:

$$\Omega = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-4x^2-9y^2}\,dxdy$$

Proposed by Jalil Hajimir-Canada

Solution 1 and generalization by Daniel Sitaru-Romania, Solution 2 by Nelson Javier Villaherrera Lopez-El Salvador

Solution 1 and generalization by Daniel Sitaru-Romania

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-4x^2-9y^2}\,dxdy = \left(\int_{-\infty}^{\infty} e^{-4x^2}dx\right)\left(\int_{-\infty}^{\infty} e^{-9y^2}dy\right) =$$

$$= 2\left(\int_{0}^{\infty} e^{-4x^2}dx\right)\cdot 2\left(\int_{0}^{\infty} e^{-9y^2}dy\right) = 2\cdot\frac{1}{2}\sqrt{\frac{\pi}{4}}\cdot 2\cdot\frac{1}{2}\sqrt{\frac{\pi}{9}} = \frac{\pi}{6}$$

GENERALIZATION:

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$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\ldots\int_{-\infty}^{\infty} e^{-x_1^2-4x_2^2-9x_3^2-\cdots-n^2x_n^2}\,dx_1dx_2\ldots dx_n =$$

$$=\prod_{k=1}^{n}\left(\int_{-\infty}^{\infty} e^{-k^2x_k^2}dx_k\right) = 2^n\prod_{k=1}^{n}\left(\int_{0}^{\infty} e^{-k^2x_k^2}dx_k\right) =$$

$$= 2^n\prod_{k=1}^{n}\left(\frac{1}{2}\sqrt{\frac{\pi}{k^2}}\right) = \prod_{k=1}^{n}\left(\sqrt{\frac{\pi}{k^2}}\right) = \frac{\left(\sqrt{\pi}\right)^n}{n!}$$

Solution 2 by Nelson Javier Villaherrera Lopez-El Salvador

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-4x^2-9y^2}\,dxdy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(2x)^2-(3y)^2}\,dxdy =$$

$$=\frac{1}{2}\cdot\frac{1}{3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x)^2-(y)^2}\,dxdy = \frac{1}{6}\int_{0}^{2\pi}\int_{0}^{\infty} e^{-\rho^2}d\rho d\varphi = \frac{-\pi}{6}\lim_{\rho\to\infty}\left(e^{-\rho^2}-1\right) = \frac{\pi}{6}$$