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**If $0 < a \le b < \frac{\pi}{2}, a + b = \frac{\pi}{2}$ then:**

$$\int_a^b \left(\frac{\sin x}{x}\right) dx \ge \frac{b-a}{2}$$

***Proposed by Daniel Sitaru – Romania***

***Solution by Marian Dincă – Romania***

***Use Hermite – Hadamard inequality:***

$f\left(\frac{a+b}{2}\right) \le \frac{\int_a^b f(x)\,dx}{b-a} \le \frac{f(a)+f(b)}{2}$, *for any convex function because* $f(x) = \frac{\sin x}{x}$, *is concave,*

$x \in \left(0, \frac{\pi}{2}\right)$; *result -* $f(x)$, *convex. We obtain:* $\frac{\sin\left(\frac{a+b}{2}\right)}{\frac{a+b}{2}} \ge \frac{\int_a^b \frac{\sin x}{x} dx}{b-a} \ge \frac{\frac{\sin a}{a} + \frac{\sin b}{b}}{2}$

*and use the well-known Jordan inequality:* $\frac{\sin x}{x} \ge \frac{2}{\pi}$, *it follows:*

$\frac{\sin a}{a} + \frac{\sin b}{b} > \frac{2}{\pi} + \frac{2}{\pi} = \frac{4}{\pi} > 1$, *it follows:*

$$\int_a^b \frac{\sin x}{x} dx \ge (b-a)\left(\frac{\frac{\sin a}{a} + \frac{\sin b}{b}}{2}\right) \ge \frac{b-a}{2}$$