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$$f(x)=\frac{\sin(\cos x)+\log x+672x^2-1341x}{x-2}$$

Find:

$$\Omega=\lim_{x\to\infty}\left(\frac{f\left(\frac{x}{672}\right)\cdot f(x)}{x}-672x\right)$$

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$$f(x)=\frac{\sin(\cos x)+\log x+672x^2-1341x}{x-2}\quad (*)$$

From the expression of $f(x)$:

$$f(x)\cdot(x-2)=\sin(\cos x)+\log x+672x^2-1341x$$

Let be the function: $g(x)=\frac{f\left(\frac{x}{672}\right)f(x)-672x^2}{x}$

$$g(x)=\frac{f\left(\frac{x}{672}\right)f(x)+\sin(\cos x)+\log x-1341x-f(x)(x-2)}{x}$$

$$g(x)=\frac{\left[f\left(\frac{x}{672}\right)-(x-2)\right]f(x)+\sin(\cos x)+\log x-1341x}{x}$$

$$g(x)=\left[\frac{\left[f\left(\frac{x}{672}\right)-(x-2)\right]f(x)}{x}+\frac{\sin(\cos x)}{x}+\frac{\log x}{x}\right]-1341$$

$$g(x)=\left[f\left(\frac{x}{672}\right)-(x-2)\right]\cdot\frac{f(x)}{x}+\frac{\sin(\cos x)}{x}+\frac{\log x}{x}-1341$$

By using Sandwich Theorem, we can easily prove that:

$$\lim_{x\to+\infty}\left(\frac{\sin(\cos x)}{x}\right)=0, \textit{ and we know } \lim_{x\to\infty}\left[\frac{\log x}{x}\right]=0$$

But: $f\left(\frac{x}{672}\right)-x+2=\frac{\sin\left(\cos\frac{x}{672}\right)+\log\left(\frac{x}{672}\right)+\frac{x^2}{672}-\frac{1341x}{672}-\frac{x^2}{672}+\frac{1346}{672}-4}{\frac{x}{672}-2}$

So: $\lim_{x\to+\infty}\left[f\left(\frac{x}{672}\right)-x-2\right]=5, \frac{f(x)}{x}=\frac{\sin(\cos x)+\log x+672x^2-1341x}{x^2-2x}$

$$\lim_{x\to+\infty}\left[\frac{f(x)}{x}\right]=672, \textit{ so:}$$

$$\lim_{x\to+\infty}g(x)=5\cdot(672)-1341=3360-1341$$

$$\lim_{x\to+\infty}g(x)=2019$$