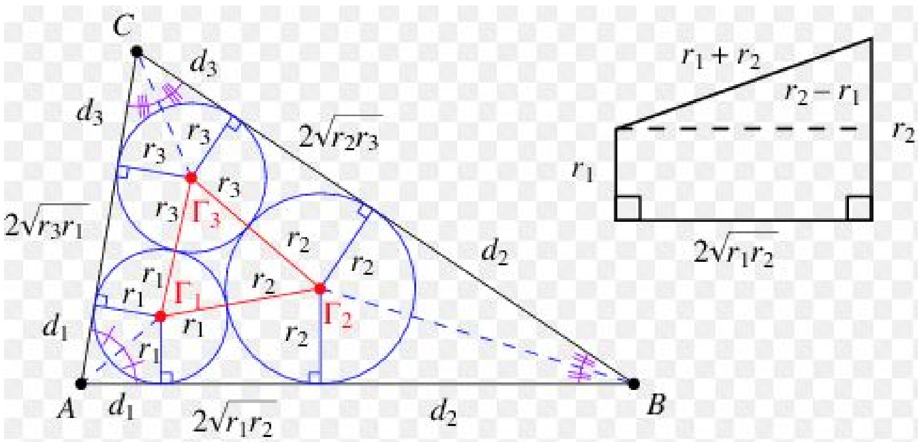

If in $\Delta ABC, r_1, r_2, r_3$ – radii of Malfatti's circles then:

$$\frac{r_1^4}{r_2^3}+\frac{r_2^4}{r_3^3}+\frac{r_3^4}{r_1^3}\geq \frac{3r}{2}\sqrt[3]{\left(1+\tan\frac{A}{2}\right)\left(1+\tan\frac{B}{2}\right)\left(1+\tan\frac{C}{2}\right)}$$

Proposed by Daniel Sitaru – Romania

Solution 1 by Soumava Chakraborty-Kolkata-India, Solution 2 by Tran Hong-Vietnam

Solution 1 by Soumava Chakraborty-Kolkata-India

$$r_1\overset{(1)}{=}\frac{\left(1+\tan\frac{B}{2}\right)\left(1+\tan\frac{C}{2}\right)}{1+\tan\frac{A}{2}}\left(\frac{r}{2}\right);r_2\overset{(2)}{=}\frac{\left(1+\tan\frac{C}{2}\right)\left(1+\tan\frac{A}{4}\right)}{1+\tan\frac{B}{4}}\left(\frac{r}{2}\right),$$

$$r_3\overset{(3)}{=}\frac{\left(1+\tan\frac{A}{4}\right)\left(1+\tan\frac{B}{4}\right)}{1+\tan\frac{C}{4}}\left(\frac{r}{2}\right)$$

$$\textit{Now, LHS}\overset{Radon}{\geq}\frac{(r_1+r_2+r_3)^4}{(r_1+r_2+r_3)^3}=r_1+r_2+r_3$$

$$\overset{by\ (1),(2),(3)}{=}\frac{r}{2}\sum\frac{\left(1+\tan\frac{B}{2}\right)\left(1+\tan\frac{C}{2}\right)}{1+\tan\frac{A}{4}}\overset{A-G}{\geq}\frac{3r}{2}\sqrt[3]{\left(1+\tan\frac{A}{4}\right)\left(1+\tan\frac{B}{4}\right)\left(1+\tan\frac{C}{4}\right)}$$

(proved)

Solution 2 by Tran Hong-Dong Thap-Vietnam

Let $x = r_1; y = r_2; z = r_3$***; Since:*** $r_1r_2r_3 = \frac{r^3}{8}\cdot\prod\left(1+\tan\frac{A}{4}\right)$

$$\Rightarrow xyz = \frac{r^3}{8}\prod\left(1+\tan\frac{A}{4}\right) \Rightarrow RHS = \frac{3r}{2}\sqrt[3]{\prod\left(1+\tan\frac{A}{4}\right)} = \frac{3r}{2}\cdot\sqrt[3]{\frac{8xyz}{r^3}} =$$

$$= \frac{3r}{2}\cdot\frac{2}{r}\sqrt[3]{xyz} = 3\sqrt[3]{xyz}$$

$$LHS \overset{AM-GM}{\geq} 3\sqrt[3]{\frac{(xyz)^4}{(xyz)^3}} = 3\sqrt[3]{xyz} = RHS$$

(Proved)