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In acute ΔABC the following relationship holds:

$$s_a(r+r_a)+s_b(r+r_b)+s_c(r+r_c)\le 4(R+r)^2$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\sum s_a(r+r_a)\le 4(R+r)^2$$

$$\because s_a=\frac{2bc}{b^2+c^2}m_a\therefore s_a\le m_a\le R(1+\cos A)\Rightarrow s_a\overset{(1)}{\le}R(1+\cos A)$$

Similarly, $s_b\overset{(2)}{\le}R(1+\cos B)$ ***and*** $s_c\overset{(3)}{\le}R(1+\cos C)$

(1)+(2)+(3)$\Rightarrow\sum s_a(r+r_a)\le\sum\left\{R(1+\cos A)\left(\frac{\Delta}{s}+\frac{\Delta}{s-a}\right)\right\}$

$$=2rsR\sum\left\{\frac{s(s-a)}{bc}\cdot\frac{b+c}{s(s-a)}\right\}=2Rrs\sum\frac{a(b+c)}{abc}=\frac{4Rrs}{4Rrs}\sum ab$$

$$=s^2+4Rr+r^2\overset{Gerretsen}{\le}4R^2+4Rr+3r^2+4Rr+r^2=4(R+r)^2$$ ***(proved)***