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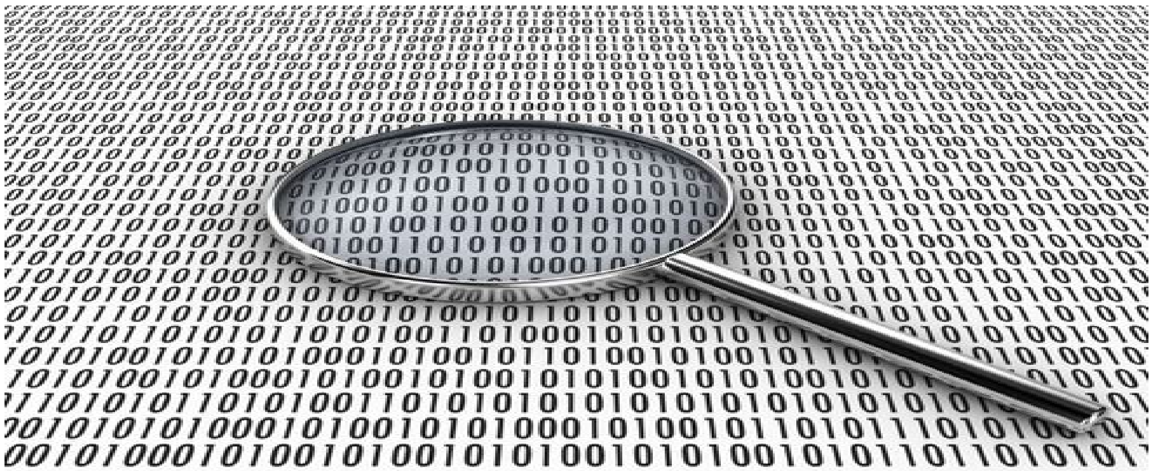

**For $n \in \mathbb{N}^*$ and $n \geq 2 \wedge \alpha_k, x_k \in \mathbb{N}^*, \forall k = \overline{1;n}$. Prove that:**

$$\sum_{k=1}^{n} \alpha_k x_k^{11} \equiv \sum_{k=1}^{n} \alpha_k, x_k \,(\text{mod } 22)$$

*Proposed by Nguyen Van Nho-Nghe An-Vietnam*

*Solution 1 by Ravi Prakash-New Delhi-India, Solution 2 by Sanong Huayrerai-Nakon Pathom-Thailand*

*Solution 1 by Ravi Prakash-New Delhi-India*

*By Fermat's little theorem,*

$$x_k^{11} \equiv x_k \,(\text{mod } 11)$$

*[$\because$ 11 is prime]*

*Also,* $x_k^{11} - x_k \equiv 0[\text{mod } 2]$

*[$\because$ $x_k$ is odd $\Rightarrow$ $x_k^{11}$ is odd and $x_k$ is even $\Rightarrow$ $x_k^{11}$ is even]*

$$\therefore 11 | (x_k^{11} - x_k) \text{ and } 2 | (x_k^{11} - x_k)$$

*As* $\gcd(11, 2) = 1$,

$$22 | (x_k^{11} - x_k)$$

$$\Rightarrow x_k^{11} \equiv x_k (\text{mod } 22)$$

$$\Rightarrow \sum_{k=1}^{n} a_k x_k^{11} \equiv \sum_{k=1}^{n} a_k x_k \,(\text{mod } 11)$$

**Solution 2 by Sanong Huayrerai-Nakon Pathom-Thailand**

*Since* $11$ *is prime*

*Hence* $a^{10} \equiv 1(\text{mod } 11), a \in \mathbb{N}\ 11 \nmid a, (11, a) = 1$

*Consider if* $\alpha_k$ *and* $x_k$ *is odd,* $k = 1, \dots n$*, let* $11i, i \in \mathbb{N}$

*Hence* $22|(x_k^{10} - 1) \to 22|\alpha_k x_k(x_k^{10} - 1) \leftrightarrow \alpha_k x_k^{11} \equiv \alpha_k x_k \ \ (\text{mod } 22)$

*2. If* $\alpha_k$ *is odd and* $x_n$ *is even,* $k = 1, \dots, n, n \neq j, j \in \mathbb{N}$ *we see that* $2|\alpha_k x_k$ *and* $11|\ (x_k^{10} - 1)$

*Hence* $22|\alpha_k x_k(x_k^{10} - 1) \leftrightarrow \alpha_k x_k^{11} = \alpha_k x_k(\text{mod } 22)$

*Form 1 and 2 we can conclude that:*

$$\sum_{k=1}^{n} \alpha_k x_k^{11} = \sum \alpha_k x_k\ (\text{mod } 22)$$

*Remark: we see that if* $\alpha_k = 11i$ *or* $x_k = 11j, j, j \in \mathbb{N}$ *then it is true.*

*Because 1* $22|\ (11i)x_k(x_k^{10} - 1)$ *and* $22|\alpha_k(11j)(11ji^{10} - 1)$

*Therefore it is true.*