PROPOSED PROBLEM

PEDRO H.O PANTOJA
UNIVERSITY OF CAMPINA GRANDE, BRAZIL

Find all positive integers a, b, c where a and b are prime numbers with $a \equiv 0 \pmod{c}$ such that $51a + 7ab + bc^2 = abc^2$.

Solution by Khanh Hung Vu - Vietnam.

$$51a + 7ab + bc^2 = abc^2 \tag{1}$$

We have $a \equiv 0 \pmod{c}$ and a is prime number, so we have two cases:
Case 1. $c = 1$
Substitute $c = 1$ for (1), we have equation (1) equivalent to:
$51a + 7ab + b = ab \Rightarrow 51a + 6ab + b = 0$ (Absurd since $a > 0$ and $b > 0$)
Case 2. $c = a$
Substitute $c = a$ for (1), we have equation (1) equivalent to:

$$51a + 7ab + ba^2 = ba^3 \Rightarrow 51 + 7b + ab = ba^2 \Rightarrow 51 = (a^2 - a - 7)b \tag{2}$$

Since b is a prime number, so (2) $\Rightarrow \begin{cases} a^2 - a - 7 = 3 \\ b = 17 \end{cases}$ or $\begin{cases} a^2 - a - 7 = 17 \\ b = 3 \end{cases}$

$\Rightarrow \begin{cases} a = \frac{1+\sqrt{41}}{2} \\ b = 17 \end{cases}$ or $\begin{cases} a = \frac{1+\sqrt{97}}{2} \\ b = 3 \end{cases}$ (Absurd since a is a prime number).

So, equation (1) has no solution satisfying the problem. □

Mathematics Department, National Economic College "Theodor Costescu", Drobeta Turnu - Severin, Mehedinti, Romania
Email address: dansitaru63@yahoo.com