PROPOSED PROBLEM

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5525. Find real values for x and y such that:

$$4\sin^2(x+y) = 1 + 4\cos^2 x + 4\cos^2 y$$

Solution by Marian Ursărescu - "Roman Vodă College" - Roman - Romania.

$$4\sin^2(x+y) = 1 + 4 - 4\sin^2 x + 4 - 4\sin^2 y \Leftrightarrow$$
$$9 - 4\sin^2 x - 4\sin^2 y - 4\sin^2(x+y) = 0 \Leftrightarrow$$
$$9 - 4\Big(\frac{1-\cos^2 x}{2}\Big) - 4\Big(\frac{1-\cos^2 y}{2}\Big) - 4\Big(\frac{1-\cos 2(x+y)}{2}\Big) = 0 \Leftrightarrow$$
$$9 - 6 + 2(\cos 2x + \cos 2y + \cos 2(x+y)) = 0 \Leftrightarrow$$
$$3 + 2[2\cos(x+y)\cos(x-y) + 2\cos^2(x+y) - 1] = 0 \Leftrightarrow$$
$$1 + 4\cos(x+y)\cos(x-y) + 4\cos^2(x+y) = 0 \Leftrightarrow$$
$$4\cos^2(x+y) + 4\cos(x+y)\cos(x-y) + \cos^2(x-y) + \sin^2(x-y) = 0 \Rightarrow$$
$$\Leftrightarrow (2\cos(x+y) + \cos(x-y))^2 + \sin^2(x-y) = 0 \Rightarrow$$
$$\Rightarrow \begin{cases} \sin(x-y) = 0 \\ 2\cos(x+y) + \cos(x-y) = 0 \end{cases} \Rightarrow x - y = k\pi, k \in \mathbb{Z}$$

1. If $x-y = 2k\pi \Rightarrow \cos(x-y) = 1 \Rightarrow \cos(x+y) = -\frac{1}{2} \Rightarrow x+y = \pm\frac{2\pi}{3} + 2p\pi, k, p \in \mathbb{Z}$

$$\begin{cases} x - y = 2k\pi \\ x + y = \pm\frac{2\pi}{3} + 2p\pi \end{cases} \Rightarrow \begin{matrix} x = k\pi + p\pi \pm \frac{\pi}{3} \Rightarrow x = n\pi \pm \frac{\pi}{3}; n = k + p \\ y = p\pi - k\pi \pm \frac{\pi}{3} \Rightarrow y = n'\pi \pm \frac{\pi}{3}; n' = p - k \end{matrix}$$

2. If $x - y = (2k+1)\pi \Rightarrow \cos(x-y) = -1 \Rightarrow \cos(x+y) = \frac{1}{2} \Rightarrow$
$\Rightarrow x + y = \pm\frac{\pi}{3} + 2p\pi; k, p \in \mathbb{Z}$

$$\begin{cases} x - y = (2k+1)\pi \\ x + y = \pm\frac{\pi}{3} + 2p\pi \end{cases} \Rightarrow \begin{matrix} x = (k+p)\pi + \frac{\pi}{2} + \frac{\pi}{6} \\ y = (p-k)\pi - \frac{\pi}{3} \pm \frac{\pi}{6} \end{matrix}, k, p \in \mathbb{Z}$$

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