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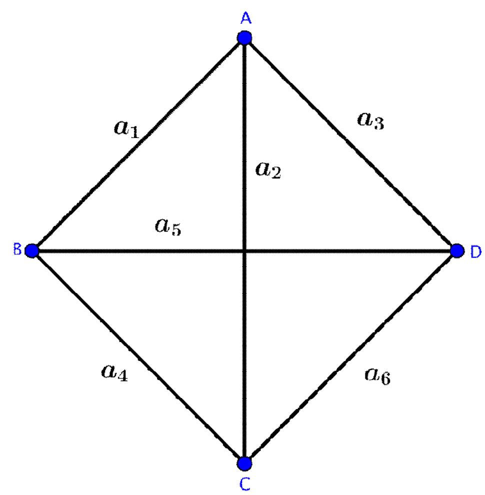
UP.162. If $ABCD$ tetrahedron; $AB = a_1; AC = a_2; AD = a_3; BC = a_4;$ $BD = a_5; CD = a_6$ then:

$$\sum_{1\le i<j\le 6}(a_i+a_j)^2 \ge 4\sqrt{3}S[ABCD]$$

$S[ABCD]$ – total area of tetrahedron $ABCD$

Proposed by Daniel Sitaru – Romania

Solution by Marian Ursărescu – Romania



$(a_i+a_j)^2 \ge 3a_ia_j \Rightarrow$ *we must show:*

$$3\sum_{1\le i<j\le 6} a_ia_j \ge 12\sqrt{3}S[ABCD] \Leftrightarrow$$

$$\sum_{1\le i<j\le 6} a_ia_j \ge 4\sqrt{3}S[ABCD] \quad \textbf{(1)}$$

Now, using Gordon's inequality: in any ΔABC we have:

$$ab+bc+ac \ge 4\sqrt{3} \Rightarrow$$

$$\left.\begin{array}{l} a_1a_2+a_1a_4+a_2a_4 \ge 4\sqrt{3}S_{ABC} \\ a_2a_6+a_2a_3+a_3a_6 \ge 4\sqrt{3}S_{ACD} \\ a_1a_5+a_1a_3+a_3a_5 \ge 4\sqrt{3}S_{ABD} \\ a_4a_5+a_4a_6+a_5a_6 \ge 4\sqrt{3}S_{BCD} \end{array}\right\} \Rightarrow \sum_{1\le i<j\le 6} a_ia_j \ge 4\sqrt{3}S[ABCD]$$