

# R M M

ROMANIAN MATHEMATICAL MAGAZINE  
www.ssmrmh.ro



**SP.161. Prove that the following inequalities holds for all real numbers  $a, b, c \in [0, 1]$**

$$(a) \quad (1 - a + a^2)(1 - b + b^2)(1 - c + c^2) \leq (1 - abc + a^2b^2c^2)$$

$$(b) \quad (1 - a + a^2)^2(1 - b + b^2)^2(1 - c + c^2)^2 \leq \\ \leq (1 - ab + a^2b^2)(1 - bc + b^2c^2)(1 - ca + c^2a^2)$$

**When does the equality occur?**

*Proposed by Nguyen Viet Hung – Hanoi – Vietnam*

*Solution by Ravi Prakash-New Delhi-India*

$$(1 - a + a^2)(1 - b + b^2) - (1 - ab + a^2b^2) = 1 - a + a^2 - b + ab - a^2b + b^2 - \\ - ab^2 + a^2b^2 - [1 - ab + a^2b^2] \\ = (a + b)^2 - (a + b) - ab(a + b) = (a + b)[a + b - ab - 1] = \\ = -(a + b)(1 - a)(1 - b) \leq 0$$

*Equality when  $a = b = 0$  or  $a = 1$  or  $b = 1$*

$$\therefore (1 - a + a^2)(1 - b + b^2) \leq 1 - ab + a^2b^2$$

*Equality when  $a = b = 0$  or  $a = 1$  or  $b = 1$ .*

$$\Rightarrow (1 - a + a^2)(1 - b + b^2)(1 - c + c^2) \leq (1 - ab + a^2b^2)(1 - c + c^2) \leq \\ \leq 1 - abc + a^2b^2c^2. \text{ Equality when } a = b = c = 0 \text{ or when at least two of } a, b, c \text{ are}$$

$$\text{equal to 1. Next, } (1 - a + a^2)^2(1 - b + b^2)^2(1 - c + c^2)^2 = \\ = [(1 - a + a^2)(1 - b + b^2)][(1 - b + b^2)(1 - c + c^2)][(1 - c + c^2)(1 - a + a^2)] \\ = [(1 - ab + a^2b^2) - (a + b)(1 - a)(1 - b)]$$

# R M M

## ROMANIAN MATHEMATICAL MAGAZINE

[www.ssmrmh.ro](http://www.ssmrmh.ro)

$$[(1 - bc + b^2c^2) - (b + c)(1 - b)(1 - c)]$$

$$[(1 - ca + c^2 + a^2) - (c + a)(1 - c)(1 - a)]$$

$$\leq (1 - ab + a^2b^2)(1 - bc + b^2c^2)(1 - ca + c^2a^2)$$

*with equality iff*

$$(a + b)(1 - a)(1 - b) = 0$$

$$(b + c)(1 - b)(1 - c) = 0$$

$$(c + a)(1 - a)(1 - c) = 0$$

$$\Leftrightarrow a = b = 0, c = 0 \text{ or } a = b = 0, c = 1 \text{ or } a = 1, b = c = 0 \text{ or } a = 0, b = 1, c = 0$$

$$\text{or } a = 1, b = 1 \text{ or } a = 1, c = 1 \text{ or } b = 1, c = 1$$