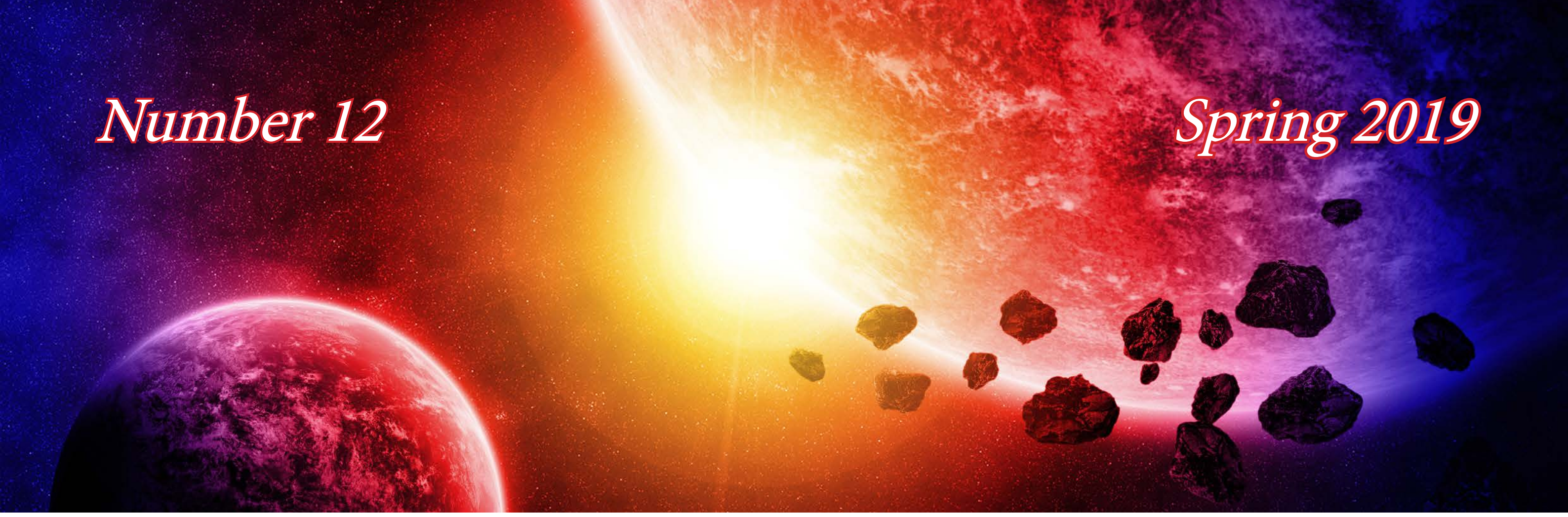

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PROBLEMS FOR JUNIORS

JP.166. If $a,b\in[0;+\infty)$ and $n\in\mathbb{N}^*\wedge n\geq 2$ then:

$$(ab)^{\frac{n}{2}}\leq\frac{\sum_{k=0}^{n}a^kb^{n-k}}{n+1}\leq\frac{a^n+b^n}{2}$$

Proposed by Nguyen Van Nho - Nghe An - Vietnam

JP.167. Let $OABC$ be a tetrahedron with $\angle AOB=\angle BOC=\angle COA=90^\circ$ and let P be any point inside the triangle ABC. Denote respectively by d_a,d_b,d_c the distances from P to faces $(OBC),(OCA),(OAB)$. Prove that:
(a) $d_a^2+d_b^2+d_c^2=OP^2$.
(b) $d_ad_bd_c\leq\frac{OA\cdot OB\cdot OC}{27}$.
(c) $OA\cdot d_a^3+OB\cdot d_b^3+OC\cdot d_c^3\geq OP^4$.

Proposed by Nguyen Viet Hung - Hanoi - Vietnam

JP.168. Let a,b,c be positive real numbers such that:

$$\frac{1}{\sqrt{1+a^3}}+\frac{1}{\sqrt{1+b^3}}+\frac{1}{\sqrt{1+c^3}}\leq 1$$

Prove that:

$$a^2+b^2+c^2\geq 12.$$

Proposed by Nguyen Viet Hung - Hanoi - Vietnam

JP.169. Let a,b,c be positive real numbers such that: $a+b+c=3$. Prove that: $\frac{a^4}{b^4\sqrt{2c(a^3+1)}}+\frac{b^4}{c^4\sqrt{2a(b^3+1)}}+\frac{c^4}{a^4\sqrt{2b(c^3+1)}}\geq\frac{a^2+b^2+c^2}{2}$

Proposed by Hoang Le Nhat Tung - Hanoi - Vietnam

JP.170. Let x,y,z be positive real numbers such that: $x+y+z=3$. Find the minimum value of:

$$P=\frac{x^4}{y^4\sqrt[3]{4z(x^5+1)}}+\frac{y^4}{z^4\sqrt[3]{4x(y^5+1)}}+\frac{z^4}{x^4\sqrt[3]{4y(z^5+1)}}$$

Proposed by Hoang Le Nhat Tung - Hanoi - Vietnam

JP.171. Let ABC be an acute triangle with perimeter 3. Prove that:

$$\frac{1}{m_a^a}+\frac{1}{m_b^b}+\frac{1}{m_c^c}\geq\frac{3}{R+r}.$$

Proposed by Nguyen Viet Hung - Hanoi - Vietnam

JP.172. Let a,b,c be positive real numbers such that: $abc=1$. Prove the inequality:

$$\frac{a^4}{b^4\sqrt{a^4+4}}+\frac{b^4}{c^4\sqrt{b^4+4}}+\frac{c^4}{a^4\sqrt{c^4+4}}\geq\sqrt{\frac{3(a+b+c)}{5}}$$

Proposed by Hoang Le Nhat Tung - Hanoi - Vietnam

JP.173. Prove that in any triangle ABC,

$$\frac{1}{\sin A}+\frac{1}{\sin B}+\frac{1}{\sin C}\geq\sqrt{\frac{6R}{r}}$$

Proposed by Nguyen Viet Hung - Hanoi - Vietnam

JP.174. Prove that in any triangle ABC,

$$\frac{h_a}{a}+\frac{h_b}{b}+\frac{h_c}{c}\geq\sqrt{6(1+\cos A\cos B\cos C)}$$

Proposed by Nguyen Viet Hung - Hanoi - Vietnam

JP.175. Prove that in any acute triangle ABC,

$$m_ar_a+m_br_b+m_cr_c\leq s^2$$

Proposed by Nguyen Viet Hung - Hanoi - Vietnam

JP.176. If $a,b>0$, then:

$$(a+b)\cdot\frac{\sin x}{x}+\frac{2ab}{a+b}\cdot\frac{\tan x}{x}>\frac{4\sqrt{2}ab}{a+b},\forall x\in\left(0;\frac{\pi}{2}\right)$$

Proposed by Rovsen Pirguliyev - Sumgait - Azebaijan

JP.177. If $a,b,c\geq 0$ then:

$$2(a+b+c)+\sum_{cyc}\sqrt{a^2+b^2-ab}\geq 3(\sqrt{ab}+\sqrt{bc}+\sqrt{ca})$$

Proposed by Daniel Sitaru - Romania

JP.178. If $a, b > 0$ then:

$$a^3 + b^3 + (\sqrt{a^2 + b^2})^3 + \frac{4a^2b^2}{a + b + \sqrt{a^2 + b^2}} > 4ab\sqrt{a^2 + b^2}$$

Proposed by Daniel Sitaru - Romania

JP.179. In acute ΔABC the following relationship holds:

$$\frac{a\cos A}{b\cos B} + \frac{b\cos B}{c\cos C} + \frac{c\cos C}{a\cos A} \leq \frac{3}{8\cos A\cos B\cos C}$$

Proposed by Daniel Sitaru - Romania

JP.180. If $a, b \geq 0$ then:

$$\begin{cases} 4ab \leq \sqrt{a^2 + b^2}(a + b + \sqrt{a^2 + b^2}) \\ 4ab\sqrt{a^2 + b^2} \leq (a^2 + b^2)(a + b + \sqrt{a^2 + b^2}) \end{cases}$$

Proposed by Daniel Sitaru - Romania

PROBLEMS FOR SENIORS

SP.166. Let $n \in \mathbb{N}^*$ and $a_k \in \mathbb{R}, \forall k = \overline{1; n}$. Find:

$$\Omega = \int \ln\left(\prod_{k=1}^{n}(x - a_k)\right)dx$$

$$(x > \max\{a_k | \forall k = \overline{1; n}\})$$

Proposed by Nguyen Viet Hung - Hanoi - Vietnam

SP.167. Let x, y, z be positive real numbers such that: $xyz = 1$. Prove that:

$$\frac{x}{\sqrt{2(x^4 + y^4)} + 4xy} + \frac{y}{\sqrt{2(y^4 + z^4)} + 4yz} + \frac{z}{\sqrt{2(z^4 + x^4)} + 4zx} +$$

$$+ \frac{2(x + y + z)}{3} \geq \frac{5}{2}$$

Proposed by Hoang Le Nhat Tung - Hanoi - Vietnam

SP.168. Let x, y, z be positive real numbers. Find the minimum possible value of:

$$\frac{x}{y + z} + \frac{y}{z + x} + 2\sqrt{\frac{1}{2} + \frac{z}{x + y}}$$

Proposed by Nguyen Viet Hung - Hanoi - Vietnam

SP.169. Prove that for all non-negative real numbers a, b, c

$$\sqrt{\frac{a^2+2}{b+c+1}}+\sqrt{\frac{b^2+2}{c+a+1}}+\sqrt{\frac{c^2+2}{a+b+1}}\geq 3$$

Proposed by Nguyen Viet Hung - Hanoi - Vietnam

SP.170. Let a, b, c, d be positive real numbers such that $a+b+c+d=2$. Prove that:

$$\frac{a}{\sqrt{b+\sqrt[3]{cda}}}+\frac{b}{\sqrt{c+\sqrt[3]{dab}}}+\frac{c}{\sqrt{d+\sqrt[3]{abc}}}+\frac{d}{\sqrt{a+\sqrt[3]{bcd}}}\geq 2$$

Proposed by Nguyen Viet Hung - Hanoi - Vietnam

SP.171. Let a, b, c be positive real numbers such that: $abc=1$. Find the minimum value of:

$$P=\frac{a^4}{b^5\sqrt{5(a^4+4)}}+\frac{b^4}{c^5\sqrt{5(b^4+4)}}+\frac{c^4}{a^5\sqrt{5(c^4+4)}}$$

Proposed by Hoang Le Nhat Tung - Hanoi - Vietnam

SP.172. Prove that for any real numbers x, y, z:

$$(x+y+z)(y+z-x)(z+x-y)(x+y-z)\leq 4y^2z^2.$$

Proposed by Nguyen Viet Hung - Hanoi - Vietnam

SP.173. Prove that for any positive real numbers x, y, z:

$$\frac{x^2\sqrt{y^2+z^2}+y^2\sqrt{z^2+x^2}+z^2\sqrt{x^2+y^2}}{x^3+y^3+z^3}\leq\sqrt{2}.$$

Proposed by Nguyen Viet Hung - Hanoi - Vietnam

SP.174. Prove that for any positive real numbers a, b, c, x, y, z:

$$(a^3+3x^3)(b^3+3y^3)(c^3+3z^3)\geq(ayz+bzx+cxy+xyz)^3$$

Proposed by Nguyen Viet Hung - Hanoi - Vietnam

SP.175. Let x, y, z be positive real numbers such that: $x^2+y^2+z^2+2xyz=1$. Find the minimum value of:

$$P=\frac{x^3}{1+3y-2yz}+\frac{y^3}{1+3z-2zx}+\frac{z^3}{1+3x-2xy}$$

Proposed by Hoang Le Nhat Tung - Hanoi - Vietnam

SP.176. Prove that if $m \in [0,\infty), x,y,z,t \in (0,\infty)$, then in any triangle ABC, with the usual notations holds:

$$\sum_{cyc} \frac{(xm_a^2 + ym_b^2)^{m+1}}{(zb^2 + tw_c^2)^m} \geq \frac{3^{m+\frac{3}{2}}(x+y)^{m+1}}{(4z+3t)^m} S.$$

Proposed by D.M. Bătineţu-Giurgiu, Neculai Stanciu - Romania

SP.177. Prove that if $m \in [0,\infty), x,y,z,t \in (0,\infty)$, then in any triangle ABC, with the usual notations holds:

$$\sum_{cyc} \frac{(xa^2 + ym_b^2)^{m+1}}{(zh_c^2 + th_a^2)^m} \geq \frac{(4x+3y)^{m+1}}{3^{m-\frac{1}{2}}(z+t)^m} S.$$

Proposed by D.M. Bătineţu-Giurgiu, Neculai Stanciu - Romania

SP.178. Prove that if $m \in [0,\infty), x,y,z,t \in (0,\infty)$, then in any triangle ABC, with the usual notations holds:

$$\sum_{cyc} \frac{(xa^2 + ym_b^2)^{m+1}}{(zm_c^2 + tm_a^2)^m} \geq \frac{(4x+3y)^{m+1}}{3^{m-\frac{1}{2}}(z+t)^m} S$$

Proposed by D.M. Bătineţu-Giurgiu, Neculai Stanciu - Romania

SP.179. If $x \in [0,1)$ then:

$$\cos x \leq x^3 + \tan^3 x + \sin^{-1} x + e^x$$

Proposed by Seyran Ibrahimov - Maasilli - Azerbaidian

SP.180. If $x,y,z \in \mathbb{R}^+ \wedge x^2+y^2+z^2 = 3^n, n \in \mathbb{N}$ then:

$$\sqrt[4]{x+y} + \sqrt[4]{x+z} + \sqrt[4]{y+z} \leq \sqrt[4]{54\sqrt{3^{n+1}}}$$

Proposed by Seyran Ibrahimov - Maasilli - Azerbaidian

UNDERGRADUATE PROBLEMS

UP.166. Solve the equation in $\mathbb{R}$:

$$\sqrt{x^3 - 2x^2 + 2x} + 3\sqrt[3]{x^2 - x + 1} + 2\sqrt[4]{4x - 3x^4} = \frac{x^4 - 3x^3}{2} + 7$$

Proposed by Hoang Le Nhat Tung - Hanoi - Vietnam

UP.167. Let a, b, c be positive real numbers such that: $abc = 1$. Find the maximum value of:

$$P = \frac{1}{\sqrt[3]{3a^4 - 4a + 2b^2 + 11}} + \frac{1}{\sqrt[3]{3b^4 - 4b + 2c^2 + 11}} + \\ + \frac{1}{\sqrt[3]{3c^4 - 4c + 2a^2 + 11}}$$

Proposed by Hoang Le Nhat Tung - Hanoi - Vietnam

UP.168. Let be $a > 0$ and $f : (-\infty, -a-1) \cup (-a, +\infty) \to \mathbb{R}$; $f(x) = \frac{1}{x^2+(2a+1)x+a^2+a}$. Find:

$$\lim_{n\to\infty} \sqrt[n^2]{\left| \lim_{p\to\infty} \sum_{k=1}^{p} f^{(n)}(k) \right|}$$

Proposed by Marian Ursărescu - Romania

UP.169. Let be the sequence $x_1 > 0$ and $x_1^p + x_2^p + \ldots + x_n^p = \frac{1}{p+\sqrt[1]{x_{n+1}}}, \forall n \in \mathbb{N}, p \in \mathbb{N}^*$. Find:

$$\lim_{n\to\infty} n^{p+1} \cdot x_n^{p^2+p+1}.$$

Proposed by Marian Ursărescu - Romania

UP.170. Find:

$$\lim_{n\to\infty} \int_0^1 \frac{\arctan(nx)\ln(1+x)}{1+x^2} dx$$

Proposed by Marian Ursărescu - Romania

UP.171. Find that in any acute-angled ΔABC the following inequality holds:

$$\min\left(\frac{\sin A}{\sin B + \sin C}, \frac{\sin B}{\sin A + \sin C}, \frac{\sin C}{\sin A + \sin B}\right) \le \frac{\cos A + \cos B + \cos C}{3} \le \\ \le \max\left(\frac{\sin A}{\sin B + \sin C}, \frac{\sin B}{\sin A + \sin C}, \frac{\sin C}{\sin A + \sin B}\right)$$

Proposed by Marian Ursărescu - Romania

UP.172. Let be $A \in M_5(\mathbb{R})$, invertible such that: $\det(A^2 + I_5) = 0$. Prove that:

$$\mathrm{Tr}\, A^* = 1 + \det A \cdot \mathrm{Tr}\, A^{-1}$$

Proposed by Marian Ursărescu - Romania

UP.173. Find:

$$\Omega = \lim_{n\to\infty} \sqrt[n]{6 - 2\sum_{i=2}^{n} \frac{1}{i+1}\binom{2i}{i} + 3\sum_{i=2}^{n}\binom{2i}{i}}$$

Proposed by Daniel Sitaru - Romania

UP.174. If $f : [a,b] \to [1,\infty); 0 < a \leq b; f$ integrable then:

$$\int_a^b \int_a^b \int_a^b \frac{3 + f(x) + f(y) + f(z)}{f(x)f(y) + f(y)f(z) + f(z)f(y)} dxdydz \leq (b-a)^3 + \left(\int_a^b \frac{dx}{f(x)}\right)^3$$

Proposed by Daniel Sitaru - Romania

UP.175. In acute ΔABC the following relationship holds:

$$\frac{b^2+c^2+2bc}{b^2+c^2-a^2} + \frac{c^2+a^2+2ca}{c^2+a^2-b^2} + \frac{a^2+b^2+2ab}{a^2+b^2-c^2} > 9$$

Proposed by Daniel Sitaru - Romania

UP.176. Let a, b be positive real numbers such that: $a + b = 2$. Find the minimum value of:

$$P = \frac{1}{a^3+b^3+2} + \frac{1}{ab} + \sqrt[3]{ab}$$

Proposed by Hoang Le Nhat Tung - Hanoi - Vietnam

UP.177. If $x, y, z, t > 1$ then:

$$(\log_{xzt} x)(\log_{xyt} y)(\log_{xyz} z)(\log_{yzt} t) < \frac{1}{16}$$

Proposed by Daniel Sitaru - Romania

UP.178. Let be $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}; B = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$. Find: $\Omega = e^A \cdot (e^B)^{-1}$; ($e^A$ - exponential matrix)

Proposed by Daniel Sitaru - Romania

UP.179. If in $\Delta ABC, a \geq b \geq c$ then the following relationship holds:

$$\sqrt[5]{\frac{m_a}{m_b}} + \sqrt[5]{\frac{m_b}{m_c}} + \sqrt[5]{\frac{m_c}{m_a}} - \sqrt[5]{\frac{m_a}{m_c}} - \sqrt[5]{\frac{m_b}{m_a}} - \sqrt[5]{\frac{m_c}{m_b}} < 1$$

Proposed by Daniel Sitaru - Romania

UP.180. If $f:(0,\infty)\to(0,\infty)$ such that exists

$$\lim_{x\to\infty}\frac{f(x+1)}{xf(x)}=a>0 \text{ and exists } \lim_{x\to\infty}\frac{(f(x))^{\frac{1}{x}}}{x} \text{ then find:}$$

$$\Omega=\lim_{x\to\infty}\left((f(x))^{\frac{2}{x+1}}\cdot\Big(\frac{(f(x+1))^{\frac{1}{x+1}}}{(x+1)^2}-\frac{(f(x))^{\frac{1}{x}}}{x^2}\Big)\right)$$

Proposed by D.M. Bătineţu-Giurgiu, Neculai Stanciu - Romania

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