If $a, b, c > 0$ and $x, y, z \geq 1$ then:

$$\left(\frac{xz}{y}\right)^{2a}\left(\frac{yx}{z}\right)^{2b}\left(\frac{zy}{x}\right)^{2c} \leq x^{\frac{a^2+b^2}{c}} y^{\frac{b^2+c^2}{a}} z^{\frac{c^2+a^2}{b}}$$

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Proof.

We have: $2a + 2b - 2c \leq \dfrac{a^2+b^2}{c} \Leftrightarrow (a-c)^2 + (b-c)^2 \geq 0$ *therefore*

$$\begin{cases} (2a+2b-2c)\ln x \leq \frac{a^2+b^2}{c}\ln x \\ (2b+2c-2a)\ln y \leq \frac{b^2+c^2}{a}\ln y \\ (2c+2a-2b)\ln z \leq \frac{c^2+a^2}{b}\ln z \end{cases}$$

. After addition we obtain:

$$\sum(2a+2b-2c)\ln x = \sum 2a(\ln x - \ln y + \ln z) = \sum \ln\left(\frac{xz}{y}\right)^{2a} \leq$$

$$\leq \sum \frac{a^2+b^2}{c}\ln x = \sum \ln x^{\frac{a^2+b^2}{c}}$$

□