In all acute-angle triangle ABC holds:

$$\sum\left(\frac{\cosh\frac{A}{2}}{\cos\frac{A}{2}}\right)^2 \le \frac{s^2+r^2-4R^2}{s^2-(2R+r)^2}$$

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Proof.

If $x \in \left(0, \frac{\pi}{2}\right) \Rightarrow \cos\frac{x}{2} \le \cosh\frac{x}{2} \Rightarrow \tanh\frac{x}{2} \le \tan\frac{x}{2} \Rightarrow \cosh\frac{x}{2} \le \frac{1}{\sqrt{1-\tan^2\frac{x}{2}}} =$

$$= \frac{\cos\frac{x}{2}}{\sqrt{\cos x}} \Rightarrow \left(\frac{\cosh\frac{x}{2}}{\cos\frac{x}{2}}\right)^2 \le \frac{1}{\cos x} \Rightarrow \sum\left(\frac{\cosh\frac{A}{2}}{\cos\frac{A}{2}}\right)^2 \le \sum\frac{1}{\cos A} = \frac{s^2+r^2-4R^2}{s^2-(2R+r)^2}$$

□