R M M

ROMANIAN MATHEMATICAL MAGAZINE
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JP.006. *Prove that if* $a, b, c \in \mathbb{R}$, $a + b + c = 2$ ***then:***

$$2(a^4 + b^4 + c^4) + 10(a^2 + b^2 + c^2) > 5(a^3 + b^3 + c^3) + 1$$

Proposed by Daniel Sitaru-Romania

Solution by proposer

$$2a^4 + 10a^2 - 5a^3 - 8a + 5 =$$

$$= 2a^4 - 3a^3 + 5a^2 - 2a^3 + 3a^2 - 5a + 2a^2 - 3a + 5 =$$

$$= a^2(2a^2 - 3a + 5) - a(2a^2 - 3a + 5) + (2a^2 - 3a + 5) =$$

$$= (2a^2 - 3a + 5)(a^2 - a + 1) =$$

$$= \left[2\left(a - \frac{3}{4}\right)^2 + \frac{31}{4}\right]\left[\left(a - \frac{1}{2}\right)^2 + \frac{3}{4}\right] > 0$$

$$2a^4 + 10a^2 - 5a^3 - 8a + 5 > 0$$
$$2b^4 + 10b^2 - 5b^3 - 8b + 5 > 0$$
$$2c^4 + 10c^2 - 5c^3 - 8c + 5 > 0$$

$$2(a^4 + b^4 + c^4) + 10(a^2 + b^2 + c^2) + 1 > 5(a^3 + b^3 + c^3) + 8(a + b + c)$$

$$2(a^4 + b^4 + c^4) + 10(a^2 + b^2 + c^2) > 5(a^3 + b^3 + c^3) + 16 - 15$$

$$2(a^4 + b^4 + c^4) + 10(a^2 + b^2 + c^2) > 5(a^3 + b^3 + c^3) + 1$$