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In acute ΔABC the following relationship holds:

$$2 \sum a^2 \cos^2 A (b \cos B + c \cos C)^2 \leq \left(\sum a \cos A \right) \prod (b \cos B + c \cos C)$$

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Solution by Soumava Chakraborty-Kolkata-India

Let $x = a \cos A, y = b \cos B, z = c \cos C; x, y, z > 0$. Then given inequality becomes:

$$2x^2(y+z)^2 + 2y^2(z+x)^2 + 2z^2(x+y)^2 \leq (x+y+z)(y+z)(z+x)(x+y) \Leftrightarrow$$

$$\Leftrightarrow (x^3y + xy^3 - 2x^2y^2) + (y^3z + yz^3 - 2y^2z^2) + (z^3x + zx^3 - 2z^2x^2) = 0 \Leftrightarrow$$

$$\Leftrightarrow xy(x-y)^2 + yz(y-z)^2 + zx(z-x)^2 \geq 0 \rightarrow \text{true (Proved)}$$