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If $x, y \in \left(0, \frac{\pi}{2}\right)$ *then:*

$$\frac{(\sin^2 x + \sin^2 y)^{\sin^2 x + \sin^2 y} \cdot (\cos^2 x + \cos^2 y)^{\cos^2 x + \cos^2 y}}{(\sin x)^{2\sin^2 x} \cdot (\sin y)^{2\sin^2 y} \cdot (\cos x)^{2\cos^2 x} \cdot (\cos y)^{2\cos^2 y}} \le 4$$

Proposed by Daniel Sitaru – Romania

Solution by Soumava Chakraborty-Kolkata-India

Let $\sin^2 x = a, \sin^2 y = b, \cos^2 x = c, \cos^2 y = d$

Then, given inequality $\Leftrightarrow \frac{(a+b)^{a+b}(c+d)^{c+d}}{a^a b^b c^c d^d} \overset{(1)}{\le} 4$

Now, $\sqrt[a+b]{a^a b^b} \overset{\text{weighted } GM-HM}{\ge} \frac{a+b}{\frac{a}{a}+\frac{b}{b}} = \frac{a+b}{2} \Rightarrow a^a b^b \overset{(a)}{\ge} \frac{(a+b)^{a+b}}{2^{a+b}}$. *Similarly,* $c^c d^d \overset{(b)}{\ge} \frac{(c+d)^{c+d}}{2^{c+d}}$

(a).(b)$\Rightarrow a^a b^b c^c d^d \ge \frac{(a+b)^{a+b} \cdot (c+d)^{c+d}}{2^{a+b+c+d}} = \frac{(a+b)^{a+b}(c+d)^{c+d}}{2^{(\sin^2 x + \cos^2 x)+(\sin^2 y + \cos^2 y)}} = \frac{(a+b)^{a+b}(c+d)^{c+d}}{4} \Rightarrow$

$\Rightarrow \frac{(a+b)^{a+b}(c+d)^{c+d}}{a^a b^b c^c d^d} \le 4 \Rightarrow$ *(1) is true (Proved)*