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In ΔABC the following relationship holds:

$$\frac{a^3}{r_b + r_c} + \frac{b^3}{r_c + r_a} + \frac{c^3}{r_a + r_b} = 2[I_a I_b I_c] - 4S,$$

$\Delta I_a I_b I_c$ – excentral triangle

Proposed by Mehmet Sahin-Ankara-Turkey

Solution by Daniel Sitaru-Romania

$$\begin{aligned} \sum \frac{a^3}{r_b + r_c} &= \sum \frac{a^3}{\frac{S}{s-b} + \frac{S}{s-c}} = \frac{1}{S} \sum \frac{a^3(s-b)(s-c)}{s-c+s-b} = \\ &= \frac{1}{S} \sum a^2(s-b)(s-c) = \frac{1}{rs} \cdot 4rs^2(R-r) = 4s(R-r) = 4sR - 4S = \\ &= \frac{4Rrs}{r} - 4S = \frac{4RS}{r} - 4S = \frac{abcS}{rS} - 4S = 2 \cdot \frac{abcS}{2r^2s} - 4S = 2[I_a I_b I_c] - 4S \end{aligned}$$