R M M

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If $x, y, z \in \mathbb{R}, x + y + z = 0$ ***then:***

$$2\sqrt{2(1+e^x)(1+e^y)(1+e^z)} \geq \left(1+\frac{1}{\sqrt{e^x}}\right)\left(1+\frac{1}{\sqrt{e^y}}\right)\left(1+\frac{1}{\sqrt{e^z}}\right)$$

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Solution by Daniel Sitaru-Romania

$$\sqrt{1+e^x} \overset{QM-AM}{\geq} \frac{1}{\sqrt{2}}\left(1+\sqrt{e^x}\right) \to \prod\sqrt{1+e^x} \geq \frac{1}{2\sqrt{2}}\prod\left(1+\sqrt{e^x}\right) \leftrightarrow$$

$$2\sqrt{2}\prod\sqrt{1+e^x} \geq \frac{1}{\sqrt{e^{x+y+z}}}\cdot\prod\left(1+\sqrt{e^x}\right), (x+y+z=0) \leftrightarrow$$

$$2\sqrt{2(1+e^x)(1+e^y)(1+e^z)} \geq \prod\left(1+\frac{1}{\sqrt{e^x}}\right)$$