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If $x, y, z \geq 0, x + y + z = \frac{\pi}{4}$ then:

$$\sum \tan x (1 + \tan y) \geq 2\sqrt{\tan x \cdot \tan y \cdot \tan z}$$

Proposed by Daniel Sitaru – Romania

Solution 1 by Amit Dutta-Jamshedpur-India, Solution 2 by Serban George

Florin-Romania

Solution 1 by Amit Dutta-Jamshedpur-India

$$\rightarrow x + y + z = \frac{\pi}{4}; x + y = \frac{\pi}{4} - z; \tan(x + y) = \tan\left(\frac{\pi}{4} - z\right);$$

$$\frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{1 - \tan z}{1 + \tan z}$$

$$\begin{aligned} (\tan x + \tan y)(1 + \tan z) &= (1 - \tan z)(1 - \tan x \tan y) \Rightarrow \tan x + \tan y + \tan z + \\ &+ \tan x \tan y + \tan y \tan z + \tan x \tan z = 1 + \tan x \tan y \tan z \Rightarrow \\ &\Rightarrow \sum \tan x (1 + \tan y) = 1 + \tan x \tan y \tan z. \end{aligned}$$

$$\text{Using AM-GM: } \frac{1 + \tan x \tan y \tan z}{2} \geq \sqrt{\tan x \cdot \tan y \cdot \tan z} \Rightarrow 1 + \tan x + \tan y \tan z \geq$$

$$\geq 2\sqrt{\tan x \tan y \tan z} \Rightarrow \sum \tan x (1 + \tan y) \geq 2\sqrt{\tan x \tan y \tan z}$$

Solution 2 by Serban George Florin-Romania

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$$\tan x = a, \tan y = b, \tan z = c$$

$$\tan(x + y + z) = \tan \frac{\pi}{4} = 1 = \frac{\sum \tan x - \prod \tan x}{1 - \sum \tan x \tan y} \Rightarrow$$

$$a + b + c - abc = 1 - ab - bc - ac$$

$$\Rightarrow a + b + c + ab + bc + ac = 1 + abc$$

$$\begin{aligned} \sum \tan x (1 + \tan y) &= \sum a(1 + b) = a + b + c + ab + bc + ac = 1 + abc \stackrel{(M_a \geq M_g)}{\geq} \\ &\geq 2\sqrt{1 \cdot abc} = 2\sqrt{abc} \end{aligned}$$