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If $a, b, c > 0, a + b + c = 3$ then:

$$\frac{\sqrt{2}}{2}(\sqrt{a} + \sqrt{b} + \sqrt{c} + 3) \geq \sqrt{a+b} + \sqrt{b+c} + \sqrt{c+a}$$

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Solution by Daniel Sitaru-Romania

$$f(x) = x^{\frac{1}{2}}, f''(x) = \frac{1}{4}x^{-\frac{3}{2}} > 0, f: (0, \infty) \rightarrow \mathbb{R}, f - \text{convexe}$$

$$\frac{1}{3} \sum f(a) + f\left(\frac{a+b+c}{3}\right) \geq \frac{2}{3} \sum f\left(\frac{a+b}{2}\right) \rightarrow \frac{1}{3} \sum \sqrt{a} + f\left(\frac{3}{3}\right) \geq \frac{2}{3} \sum \sqrt{\frac{a+b}{2}} \rightarrow$$

$$\begin{aligned} \sum \sqrt{a} + 3f(1) &\geq \frac{2}{\sqrt{2}} \sum \sqrt{a+b} \rightarrow \frac{\sqrt{2}}{2}(\sqrt{a} + \sqrt{b} + \sqrt{c} + 3) \\ &\geq \sqrt{a+b} + \sqrt{b+c} + \sqrt{c+a} \end{aligned}$$