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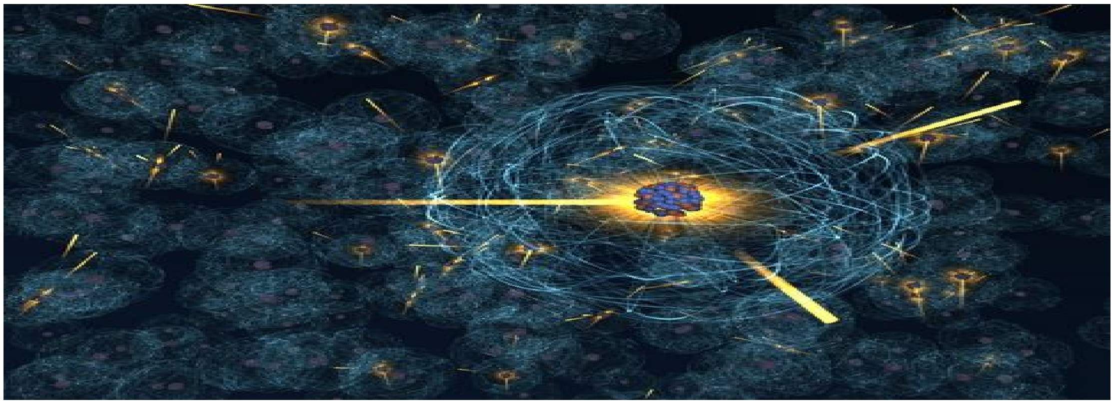

In $\Delta\, ABC$ the following relationship holds:

$$\begin{vmatrix} a & 0 & c & b \\ 0 & a & b & c \\ b & c & 0 & a \\ c & b & a & 0 \end{vmatrix} \geq 432r^4$$

Proposed by Daniel Sitaru – Romania

Solution by Soumava Chakraborty-Kolkata-India

$$LHS = a \times \begin{vmatrix} a & b & c \\ c & 0 & a \\ b & a & 0 \end{vmatrix} + c \times \begin{vmatrix} 0 & a & c \\ b & c & a \\ c & b & 0 \end{vmatrix} - b \begin{vmatrix} 0 & a & b \\ b & c & 0 \\ c & b & a \end{vmatrix} =$$

$$= a\{a(-a^2) - b(-ab) + c \cdot ca\} + c\{-a(-ac) + c(b^2 - c^2)\} -$$

$$-b\{-a(ab) + b(b^2 - c^2)\} = a(-a^3 + ab^2 + ac^2) + c(a^2c + b^2c - c^3) +$$

$$+b(-a^2b + b^3 - bc^2) = a^2(b^2 + c^2 - a^2) + c^2(a^2 + b^2 - c^2) + b^2(c^2 + a^2 - b^2)$$

$$= 2a^2bc\cos A + 2c^2abc\cos C + 2b^2ca\cos B = 2abc\left(\sum a\cos A\right) =$$

$$= 2Rabc(\sin 2A + \sin 2B + \sin 2C) = 2Rabc \cdot 4\sin A \sin B \sin C$$

$$= 2R \cdot 4Rrs\left(4\frac{abc}{8R^3}\right)$$

$$= 16\frac{R^2rs \cdot Rrs}{R^3} = 16r^2s^2 \overset{s \geq 3\sqrt{3}r}{\geq} 16 \cdot 27r^4 = 432r^4$$

(Proved)