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ROMANIAN MATHEMATICAL MAGAZINE
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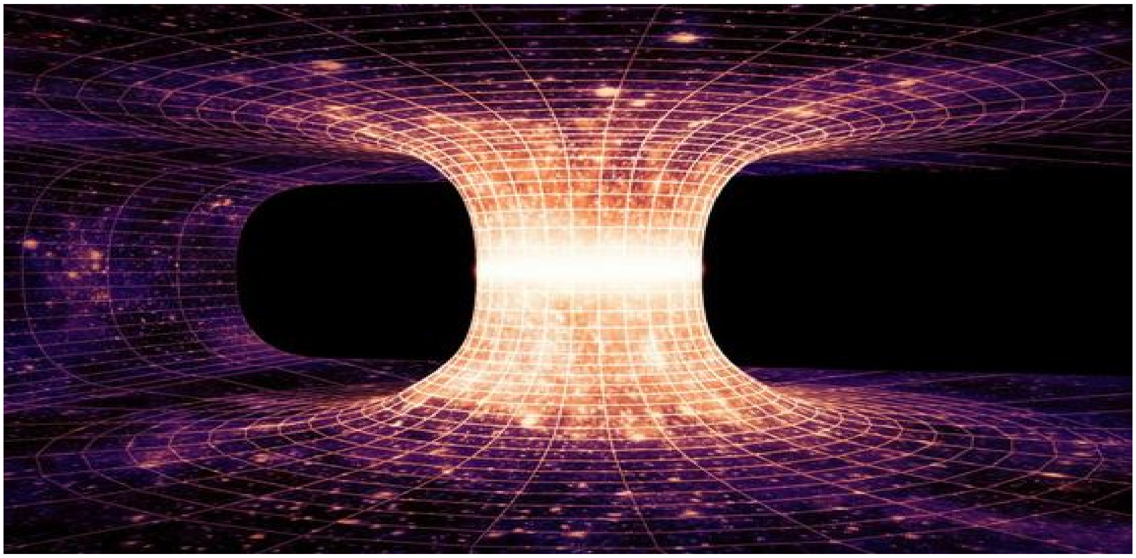

**Find:**

$$\Omega = \lim_{n\to\infty}\frac{1}{n}\left(\tan^{-1} n + \frac{1}{2}\tan^{-1}(n-1) + \frac{1}{3}\tan^{-1}(n-2) + \cdots + \frac{1}{n}\tan^{-1} 1\right)$$

*Proposed by Daniel Sitaru – Romania*

*Solution by Marian Ursărescu – Romania*

$$\text{Let } a_n = \frac{1}{n}\left(\arctan n + \frac{1}{2}\arctan(n-1) + \cdots + \frac{1}{n}\arctan 1\right)$$

$$|a_n| = \frac{1}{n}\left|\arctan n + \frac{1}{2}\arctan(n-1) + \cdots + \frac{1}{n}\arctan 1\right| \leq$$

$$\leq \frac{1}{n}|\arctan n| + \frac{1}{2}|\arctan(n-1)| + \cdots + \frac{1}{n}|\arctan 1| \leq \frac{\pi}{2}\frac{\left(1+\frac{1}{2}+\cdots+\frac{1}{n}\right)}{2} \quad (1)$$

$$\text{But } \lim_{n\to\infty}\frac{1+\frac{1}{2}+\cdots+\frac{1}{n}}{n} \overset{C.S}{=} \lim_{n\to\infty}\frac{1}{n+1} = 0 \quad (2)$$

From (1) + (2) $\Rightarrow \Omega = 0$.