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SP.116. A triangle with side lenghts a, b, c has perimeter equal to 3.

Prove that: $a^3+b^3+c^3+a^4+b^4+c^4\geq 2(a^2b^2+b^2c^2+c^2a^2)$

Proposed by George Apostolopoulos – Messolonghi – Greece

Solution by Soumava Chakraborty-Kolkata-India

$$1=\frac{\sum a}{3} \therefore \textit{given inequality} \Leftrightarrow \left(\sum a\right)\left(\sum a^3\right)+3\sum a^4\geq 6\sum a^2b^2$$

$$\Leftrightarrow \sum a^4+\sum a^3b+\sum ab^3+3\sum a^4\geq 6\sum a^2b^2$$

$$\Leftrightarrow 4\sum a^4+\sum a^3b+\sum ab^3\geq 6\sum a^2b^2 \quad (1)$$

Now, $\sum a^3b+\sum ab^3 \underset{(a)}{\overset{A-G}{\geq}} 2\sum a^2b^2$. Also, $4\sum a^4 \overset{(b)}{\geq} 4\sum a^2b^2$

(a)+(b)⇒(1) is true (Proved)