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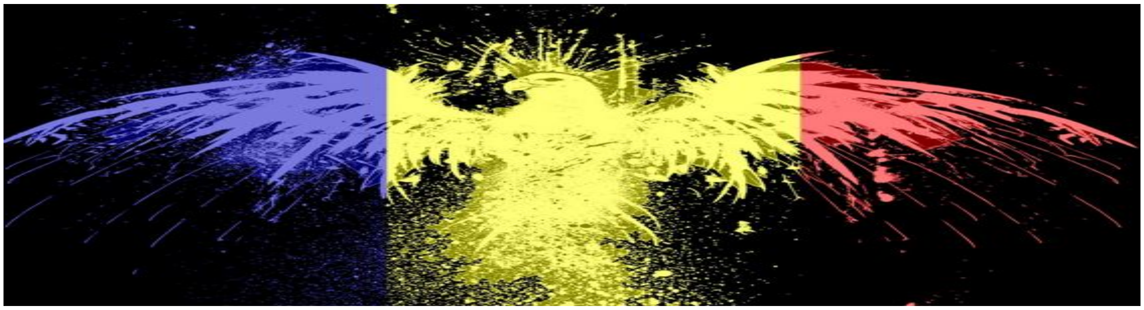

SP.107. Prove that:

$$\left(\int_0^1 arctan^2 x\,dx\right)\left(\int_0^1 \frac{dx}{arctan^2\left(\frac{1}{x^2-x+1}\right)}\right) > \frac{1}{4}$$

Proposed by Daniel Sitaru – Romania

Solution by Soumitra Mandal-Chandar Nagore-India

We know $x \geq \tan^{-1} x$ ***and*** $\tan^{-1} x \geq x + \frac{x^3}{3}$ ***for all*** $x \geq 0$

$$\left(\int_0^1 (\tan^{-1} x)^2 dx\right)\left(\int_0^1 \frac{dx}{\left(\tan^{-1}\frac{1}{x^2-x+1}\right)^2}\right)$$

$$\geq \left(\int_0^1 \left(x+\frac{x^3}{3}\right)^2 dx\right)\left(\int_0^1 (x^2-x+1)^2 dx\right)$$

$$= \left(\frac{1}{3}+\frac{1}{63}+\frac{2}{15}\right)\left(\frac{1}{5}+\frac{1}{3}+1-\frac{1}{2}-1+\frac{2}{3}\right) = \frac{152}{315}\cdot\frac{7}{10} > \frac{1}{4}$$

(Proved)