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ROMANIAN MATHEMATICAL MAGAZINE
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In $\Delta\ ABC$ the following relationship holds:

$$\left(\sum \frac{\sin B + \sin C}{\sin A}\right)\left(\sum \frac{\cos\frac{B}{2} + \cos\frac{C}{2}}{\cos\frac{A}{2}}\right) \geq 16\left(\sum \frac{\sin A}{\sin B + \sin C}\right)\left(\sum \frac{\cos\frac{A}{2}}{\cos\frac{B}{2} + \cos\frac{C}{2}}\right)$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\forall x, y, z > 0 \left(\sum \frac{y+z}{x}\right) \geq 4\left(\sum \frac{x}{y+z}\right)$$

$$\Leftrightarrow \frac{\sum yz\,(y+z)}{xyz} \geq \frac{4\sum\{x(z+x)(x+y)\}}{(x+y)(y+z)(z+x)}$$

$$\Leftrightarrow \left\{\prod (x+y)\right\}\left(\sum x^2y + \sum xy^2\right) \geq 4xyz\left\{\sum x^3 + \left(\sum xy\right)\left(\sum x\right)\right\}$$

$$\Leftrightarrow \sum x^4y^2 + \sum x^2y^4 + 2\sum x^3y^3 \overset{(a)}{\geq} 2xyz\left(\sum x^3\right) + 6x^2y^2z^2$$

Now, $\sum x^4y^2 + \sum x^2y^4 = \sum x^4(y^2+z^2)$

$$\underset{(1)}{\overset{A-G}{\geq}} \sum x^4\, 2yz = 2xyz\left(\sum x^3\right)$$

$$2\sum x^3y^3 \underset{(2)}{\overset{A-G}{\geq}} 6x^2y^2z^2$$

(1)+(2)⇒(a) is true

$$\therefore \forall x, y, z > 0 \sum \frac{y+z}{x} \overset{(b)}{\geq} 4\sum \frac{x}{y+z}$$

$$\therefore \sum \frac{\sin B + \sin C}{\sin A} \underset{(i)}{\overset{by\ (b)}{\geq}} 4\sum \frac{\sin A}{\sin B + \sin C}$$

$$\& \sum \frac{\cos\frac{B}{2} + \cos\frac{C}{2}}{\cos\frac{A}{2}} \underset{(ii)}{\overset{by\ (b)}{\geq}} 4\sum \frac{\cos\frac{A}{2}}{\cos\frac{B}{2} + \cos\frac{C}{2}}$$

(i)×(ii)⇒ given inequality is true