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If $0 < a \leq b \leq c$ then:

$$3a^2b \leq \prod \sqrt[3]{a^3 + ab\sqrt{ab} + b^3} \leq 3bc^2$$

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Solution by Soumitra Mandal-Chandar Nagore-India

We know for $x, y \geq 0$ ***then*** $x^2 + xy + y^2 \geq 3xy$ ***and*** $\frac{3}{2}(x^2 + y^2) \geq x^2 + xy + y^2$

$$\prod_{cyc} \sqrt[3]{(a^3 + ab\sqrt{ab} + b^3)}$$

$$\Rightarrow \sqrt[3]{\prod_{cyc}\left(3a^{\frac{3}{2}}b^{\frac{3}{2}}\right)} \leq \prod_{cyc} \sqrt[3]{a^3 + ab\sqrt{ab} + b^3} \leq \sqrt[3]{\frac{27}{8}\prod_{cyc}(a^3 + b^3)}$$

$$\Rightarrow 3abc \leq \prod_{cyc} \sqrt[3]{a^3 + ab\sqrt{ab} + b^3} \leq \frac{3}{2}\sqrt[3]{\prod_{cyc}(a^3 + b^3)}$$

$$\Rightarrow 3ba^2 \leq \prod_{cyc} \sqrt[3]{a^3 + ab\sqrt{ab} + b^3} \leq \frac{3}{2}\sqrt[3]{(2b^3)(2c^2)(2c^3)}[\because a \leq b \leq c]$$

$$\therefore 3a^2b \leq \prod_{cyc} \sqrt[3]{a^3 + ab\sqrt{ab} + b^3} \leq 3bc^2$$