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Prove: $\int_0^1 \sqrt{x+\sqrt{x+\sqrt{x}}}\,dx \geq \frac{2}{3}\sqrt{1+\sqrt{2}}$

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Solution by Lazaros Zachariadis-Thessaloniki-Greece

$\forall x \in [0,1]$ ***it's*** $\sqrt{x} \geq x$ "=" ***for*** $x = 0, x = 1 \Leftrightarrow \sqrt{x} + x \geq 2x$

$$\Leftrightarrow \sqrt{\sqrt{x}+x} \geq \sqrt{2}\cdot\sqrt{x} \geq \sqrt{2}\cdot x \Leftrightarrow \sqrt{\sqrt{x}+x}+x \geq \left(\sqrt{2}+1\right)\cdot x$$

$$\Leftrightarrow \sqrt{\sqrt{x+x}+x} \geq \sqrt{\left(\sqrt{2}+1\right)}\cdot\sqrt{x} \text{ so,}$$

$$\int_0^1 \sqrt{\sqrt{x+\sqrt{x}}+x}\,dx \geq \sqrt{\sqrt{2}+1}\cdot\int_0^1 \sqrt{x}\,dx = \sqrt{\sqrt{2}+1}\cdot\left[\frac{2}{3}\cdot x^{\frac{3}{2}}\right]_0^1 = \sqrt{\sqrt{2}+1}\cdot\frac{2}{3}$$