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If $0 < a \leq b$ *then :*

$$\int_a^b \int_a^b \left(\frac{e^x + e^y - 2}{\sqrt{xy}}\right)^{10} dxdy \geq 2^{10}(b-a)^2$$

Proposed by Nho Nguyen Van-Nghe An-Vietnam

Solution by Daniel Sitaru-Romania

$$\begin{cases} e^x \geq x + 1 \\ e^y \geq y + 1 \end{cases} \rightarrow e^x + e^y \geq x + y + 2 \overset{AM-GM}{\geq} 2\sqrt{xy} + 2 \rightarrow$$

$$e^x + e^y - 2 \geq 2\sqrt{xy} \rightarrow \frac{e^x + e^y - 2}{\sqrt{xy}} > 2 \rightarrow \left(\frac{e^x + e^y - 2}{\sqrt{xy}}\right)^{10} > 2^{10} \rightarrow$$

$$\int_a^b \int_a^b \left(\frac{e^x + e^y - 2}{\sqrt{xy}}\right)^{10} \geq \int_a^b \int_a^b 2^{10} dxdy = 2^{10}(b-a)^2$$