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Let ΔDEF be the contact triangle of $\Delta\ ABC$.

If $S_1 = S[AFE], S_2 = S[BDF], S_3 = S[CDE], S_0 = S[DEF]$ then:

$$\frac{1}{S_0}+\frac{1}{S_1}+\frac{1}{S_2}+\frac{1}{S_3}\geq\frac{1}{S}\cdot\frac{4R^2}{2R-r^2}$$

Proposed by Mehmet Sahin-Ankara-Turkey

Solution by Soumava Chakraborty-Kolkata-India

Let DEF be the contact triangle of $\Delta\ ABC$. If $S_1 = S[AFE], S_2 = S[BDF], S_3 = S[CDE]$,

$$S_0 = S[DEF], \text{ then, } \frac{1}{S_0}+\frac{1}{9}\sum_{i=1}^{3}\frac{1}{S_i}\overset{(1)}{\geq}\frac{1}{S}\cdot\frac{4R^2}{2Rr-r^2}$$

$$S_0=\frac{S^2}{2Rs}=\frac{rS^2}{2RS}=\frac{rS}{2R}\ \therefore\ (1)\Leftrightarrow\frac{1}{9}\sum_{i=1}^{3}\frac{1}{S_i}\geq\frac{1}{S}\cdot\frac{4R^2}{2Rr-r^2}-\frac{2R}{rS}$$

$$=\frac{1}{S}\left(\frac{4R^2}{2Rr-r^2}-\frac{2R}{r}\right)=\frac{1}{S}\cdot\frac{2R}{2R-r}\Leftrightarrow\sum_{i=1}^{3}\frac{1}{S_i}\geq\frac{1}{S}\cdot\frac{18R}{2R-r}\to\ (2).$$

$$\text{Now, }\sum_{i=1}^{3}\frac{1}{S_i}\overset{Berstrom}{\geq}\frac{9}{S_1+S_2+S_3}=\frac{9}{S[ABC]-S[DEF]}=$$

$$=\frac{9}{S-\frac{rS}{2R}}=\frac{18R}{S(2R-r)}\Rightarrow (2)\text{ is true (proved)}$$