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**If $a,b,c,d\in\mathbb{R}$ then:**

$$a+b+c+d\le\frac{1}{2}+(a+b)(c+d)+a^2+b^2+c^2+d^2$$

*Proposed by Uche Eliezer Okeke-Anambra-Nigeria*

*Solution 1 by Ravi Prakash-New Delhi-India, Solution 2 by Nho Nguyen Van-Vietnam*

*Solution 1 by Ravi Prakash-New Delhi-India*

$$(a+b+c+d-1)^2+(a-b)^2+(c-d)^2\ge 0$$

$$\Rightarrow a^2+b^2+c^2+d^2-2(a+b+c+d)+1$$

$$+2(ab+bc+cd+ad+ac+bd)+a^2+b^2-2ab+c^2+d^2-2cd\ge 0$$

$$\Rightarrow 2(a^2+b^2+c^2+d^2)-2(a+b+c+d)+2(a+b)(c+d)+1\ge 0$$

$$\Rightarrow a+b+c+d\le\frac{1}{2}+(a+b)(c+d)+a^2+b^2+c^2+d^2$$

*Solution 2 by Nho Nguyen Van-Vietnam*

*We have:*

$$a^2+b^2\overset{?}{\ge}\frac{1}{2}(a+b)^2$$

$$c^2+d^2\overset{?}{\ge}\frac{1}{2}(c+d)^2$$

So: $\frac{1}{2}+(a+b)(c+d)+a^2+b^2+c^2+d^2$

$$\ge\frac{1}{2}[(a+b)^2+2(a+b)(c+d)+(c+d)^2+1]$$

$$=\frac{1}{2}(a+b+c+d)^2+\frac{1}{2}$$

$$\overset{?}{\ge}\frac{1}{2}[2(a+b+c+d)-1]+\frac{1}{2}=a+b+c+d\ \textit{(done)}$$

$$(x^2\ge 2x-1)$$