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If $a, b, c, d, e > 0, a + b + c + d + e = 5$ then:

$$\sum \frac{b + c + d + e}{2a + b + c + d + e} \geq \frac{10}{3}$$

Proposed by Daniel Sitaru – Romania

Solution 1 by Abdelhak Maoukouf-Casablanca-Morocco, Solution 2 by Marian Dinca-Romania, Solution 3 by Hasan Bostanlik-Sarkisla-Turkey, Solution 4 by Kunihiro Chikaya-Tokyo-Japan

Solution 1 by Abdelhak Maoukouf-Casablanca-Morocco

$$\begin{aligned} \sum_{cyc} \frac{b + c + d + e}{2a + b + c + d + e} &= \sum_{cyc} \frac{5 - a}{5 + a} = \sum_{cyc} \left(\frac{10}{5 + a} - 1 \right) = \\ &= -5 + 10 \sum_{cyc} \left(\frac{1}{5 + a} \right) \stackrel{CS}{\geq} -5 + 10 \frac{5^2}{\sum_{cyc} (5 + a)} = -5 + 10 \frac{25}{25 + 5} = \frac{10}{3} \end{aligned}$$

Solution 2 by Marian Dinca-Romania

$$\begin{aligned} \sum_{cyc} \frac{b + c + d + e}{2a + b + c + d + e} &= \sum_{cyc} \frac{5 - a}{5 + a} = \sum_{cyc} \left(\frac{5 - a}{5 + a} + 1 \right) - 5 = \\ &= 10 \left(\sum_{cyc} \frac{1}{5 + a} \right) - 5 \geq 10 \cdot \frac{25}{\sum_{cyc} 5 + a} - 5 = 10 \cdot \frac{25}{30} - 5 = \frac{25}{3} - 5 = \frac{10}{3} \end{aligned}$$

Solution 3 by Hasan Bostanlik-Sarkisla-Turkey

$$\begin{aligned} \sum \frac{5 - a}{5 + a} &= \sum \left(-1 + \frac{10}{5 + a} \right) = -5 + 10 \sum \frac{1}{5 + a} \\ \frac{5}{\frac{1}{5 + a} + \frac{1}{5 + b} + \dots + \frac{1}{5 + d}} &\leq \frac{a + b + c + d + e + 25}{5} \\ \frac{5}{\sum \frac{1}{5 + a}} \leq 6 &\Rightarrow \sum \frac{1}{5 + a} \geq \frac{5}{6} \Rightarrow \geq -5 + 10 \cdot \frac{5}{6} = \frac{10}{3} \end{aligned}$$

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Solution 4 by Kunihiko Chikaya-Tokyo-Japan

$$(*) \frac{b+c+d+e}{2a+b+c+d+e} + \frac{c+d+e+a}{2b+c+d+e+a} + \frac{d+e+a+b}{2c+d+e+a+b} + \frac{e+a+b+c}{2d+e+a+b+c} + \frac{a+b+c+d}{2e+a+b+c+d} \geq \frac{10}{3}$$

$$a + b + c + d + e = 5, a, b, c, d, e \in \mathbb{R}^+$$

$$\frac{b+c+d+e}{2a+b+c+d+e} - \left(-\frac{5}{18}a + \frac{17}{18} \right) =$$

$$= \frac{5-a}{a+5} + \frac{5a-17}{18} = \frac{5(a-1)^2}{18(a+5)} \geq 0, \text{ etc.}$$

$$\text{LHS of } (*) = \frac{5-a}{a+5} + \frac{5-b}{b+5} + \frac{5-c}{c+5} + \frac{5-d}{d+5} + \frac{5-e}{e+5}$$

$$\geq -\frac{5}{18}(a+b+c+d+e) + \frac{17}{18} \times 5$$

$$= \frac{5}{18}(-5 + 17) = \frac{10}{3}$$

Equality: $a = b = c = d = e = 1$.