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Find:

$$\Omega = \lim_{n\to\infty} \frac{1}{(n+1)^2 4^n} \cdot \sum_{k=0}^{n} (2n+1-k)\binom{2n+1}{k}$$

Proposed by Daniel Sitaru – Romania

Solution by Ravi Prakash-New Delhi-India

$$a_n = \sum_{k=0}^{n} (2n+1-2k)\binom{2n+1}{k} =$$

$$= \sum_{k=0}^{n} \left[(2n+1-k)\binom{2n+1}{k} - k\binom{2n+1}{k}\right] =$$

$$= \sum_{k=0}^{n} \left[(2n+1-k)\binom{2n+1-k}{k} - k\binom{2n+1}{k}\right] =$$

$$= \sum_{k=0}^{n} (2n+1-k)\binom{2n+1-k}{k} - \sum_{k=0}^{n} k\binom{2n+1}{k} =$$

$$= \sum_{k=0}^{n} (2n+1)\binom{2n}{k} - \sum_{k=0}^{n} (2n+1)\binom{2n}{k} = 0$$

$$\Omega = \lim_{n\to\infty} \frac{1}{(n+1)^2 4^n} \cdot a_n = \lim_{n\to\infty} \frac{1}{(n+1)^2 4^n} \cdot 0 = 0$$

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$$a_n=\sum_{k=0}^{n}(2n+1-2k)\binom{2n+1}{k}$$

$$=\sum_{k=0}^{n}\left[(2n+1-k)\binom{2n+1}{2n+1-k}-k\binom{2n+1}{k}\right]$$

$$=\sum_{k=0}^{n}(2n+1-k)\binom{2n+1}{2n+1-k}-\sum_{k=0}^{n}k\binom{2n+1}{k}$$

But $k\binom{2n+1}{k}=(2n+1)\binom{2n}{k}$

and $(2n+1-k)\binom{2n+1}{2n+1-k}=(2n+1)\binom{2n}{2n-k}$

$$=(2n+1)\binom{2n}{k}$$

$$\therefore \sum_{k=0}^{n}(2n+1-2k)\binom{2n+1}{k}$$

$$=\sum_{k=0}^{n}(2n+1)\binom{2n}{k}-\sum_{k=0}^{n}(2n+1)\binom{2n}{k}$$

$$=0$$

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$$\Omega \doteq \lim_{n\to\infty}\frac{1}{(n+1)^2 4^n}(a_n)$$

$$=0$$